

PROBLEMS
ON DESCRIPTIVE GEOMETRY

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PROBLEMS
ON DESCRIPTIVE GEOMETRY

Tutorial

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P93

Подано типові задачі з нарисної геометрії. Умови задач наведено в координатно-описовому вигляді. Це сприяє швидкому опануванню спеціальною термінологією, розвитку просторового уявлення, здатності створювати геометричний образ за його описом. До кожного розділу наведено необхідний теоретичний матеріал.

Для студентів інженерних спеціальностей, які вивчають дисципліни «Нарисна геометрія» або «Інженерна графіка».

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Typical problems on descriptive geometry are included. Initial data for the problems are represented in coordinate-descriptive way. It promotes to fast operating with special terminology, development of spatial imagination, ability to create geometrical image by its description. Necessary theory is given for each type of the problems.

For students of engineering specialties who study disciplines “Descriptive Geometry” or “Engineering Graphics”.

Figures: 54. Bibliography: 10 references

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INTRODUCTION

Engineering graphics is the primary medium for developing and communicating design concepts. It covers the total field of graphical problem solving within three major areas of specialization: descriptive geometry, computer graphics and technical drawings.

Graphic representation is a basic, natural form of communication that isn't tied to a particular time or place. It is, in a sense, a universal engineering language.

Accomplishing ideas, from the simplest to the most elaborate, requires teamwork. A new product, machine, structure, or system may exist in the mind of the engineer or designer, but before it can become a reality, the idea must be communicated to many different people.

The ability to communicate design concepts quickly and clearly through graphical technical images is a key to meeting project budgets and time constraints.

Effective graphic communication is also an advantage in the global marketplace where team members may not always share a spoken or written language.

Descriptive geometry is one of the fundamental disciplines that make up the basis of an engineering education.

It deals with setting forth and justifying methods of constructing representations of three-dimensional forms in the plane, as well as methods of solving geometrical problems on the basis of given representations of these forms.

The representations constructed according to the rules of descriptive geometry enable us to visualize the shape of objects and their relative positions in space, to determine their dimensions, and to study their geometrical properties.

Moreover, descriptive geometry provides a number of practical means for engineering drawings, ensuring their clarity and accuracy, and, hence, the possibility of manufacturing the represented objects.

Finally, descriptive geometry develops the student's three-dimensional imagination by making frequent appeals to it.

Albert Einstein said: "Imagination is more important than knowledge, for knowledge is limited, whereas imagination embraces the entire world, stimulating progress, or, giving birth to evolution."

The distinctive feature of this tutorial is that initial data for the problems are represented in coordinate-descriptive way. It promotes to development of spatial imagination and ability of geometrical images creation by their descriptions that is important for 3D-models' processing in Computer Aided Design systems.

Also coordinate-descriptive way of initial data representation in the problems allows operating with the process of parameterization that plays key role in feature-based modeling conception, implemented in modern CAD-systems.

1. METHOD OF PROJECTION

Drawing is a key way to transfer technical thoughts. There are three basic demands for the images in a drawing: **reversibility, measurability, obviousness**.

Reversibility is ability to recreate the shape of an object from its image(s).

Measurability is ability to obtain information about real sizes of an object by measuring its image(s).

Obviousness is convenience of object's shape restoration from its images.

In graphic language the shape is described by projection, which is the image of the object, formed by rays of light, taken in some particular direction, from the object into a picture plane, as it appears to an observer stationed at the point, from or towards where the projection is made.

Central Projection

Depending upon the orientation of the object, location of the point of sight, and the direction of lines of sight relative to the picture plane, different types of projections, e.g., central (perspective), parallel, orthographic, axonometric, oblique, etc., can be obtained (Fig. 1.1).

The plane, on which the projection is taken, is called the plane of projection or picture plane. The point, from which the observer is assumed to view the object, is called the station point or the center of projection.

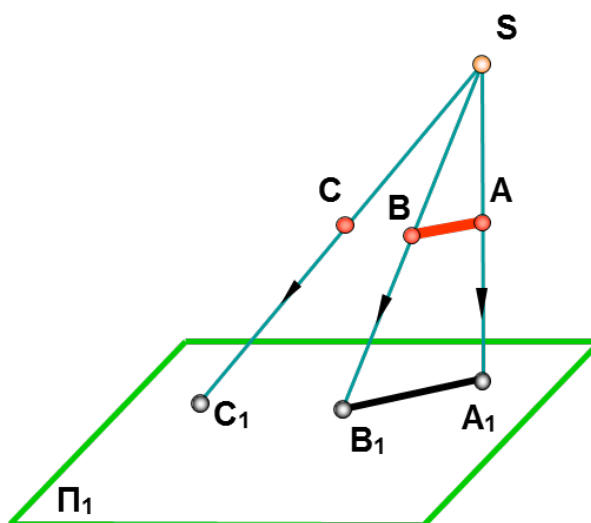


Fig. 1.1

Essentials of projection:

Π_1 – plane of projection (picture plane);

S – center of projection (station point) ($S \notin \Pi_1$);

C – a point (original);

SC – projector (projecting ray);

C_1 – projection (image) of a point C;

AB – straight line segment (original);

SA, SB – projectors (projecting rays);

A_1B_1 – central projection of AB line segment onto the plane Π_1 .

Method, shown in Fig. 1.1 is called First angle projection, where original object is placed between the center of projection and picture plane.

An algorithm to construct central projection of an object:

- From the point S draw a projecting ray through any point of a figure;
- Find the piercing point between projector and a plane of projection Π_1 . Obtained point is a projection (image) of an original to the plane of projection;
- Find projections of the points A_1 and B_1 , join them and obtain central projection of AB line segment.

Method, shown in Fig. 1.2 is called Third angle projection, where picture plane is placed between a center of projection and an object.

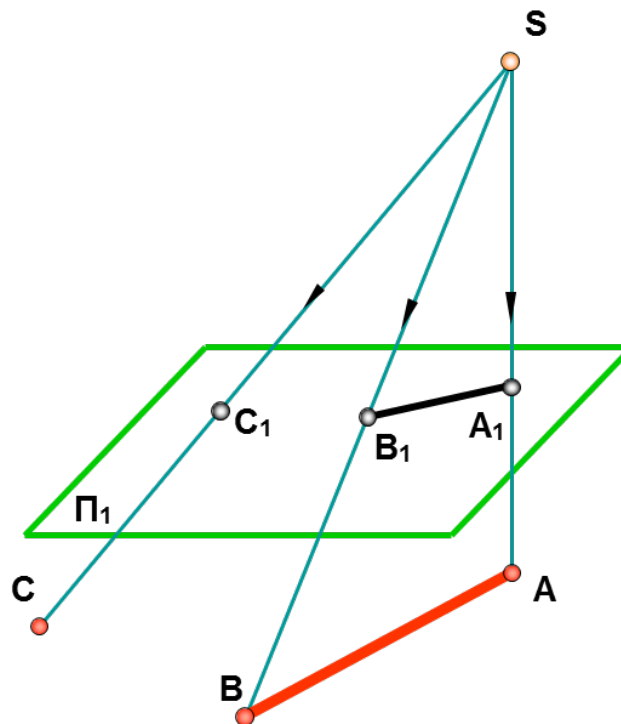


Fig. 1.2

Properties of Projection

There are four main properties of Central Projection, which help to create reversible images using projection method (Fig. 1.3):

1. Point projects to a point ($A \rightarrow A_1$).
2. Straight line projects to a straight line: ($[MK] \rightarrow [M_1K_1]$).

Exception to this property: if a straight line segment belongs to the projecting ray, then it projects to a point: ($[EF] \rightarrow E_1 = F_1$).

3. Plane projects to a plane.

Exception to this property: if a plane contains projectors, then it projects to a straight line.

4. If the point belongs to some geometrical object, then projection of that point belongs to the projection of that geometrical object ($N \in [MK] \rightarrow N_1 \in [M_1K_1]$).

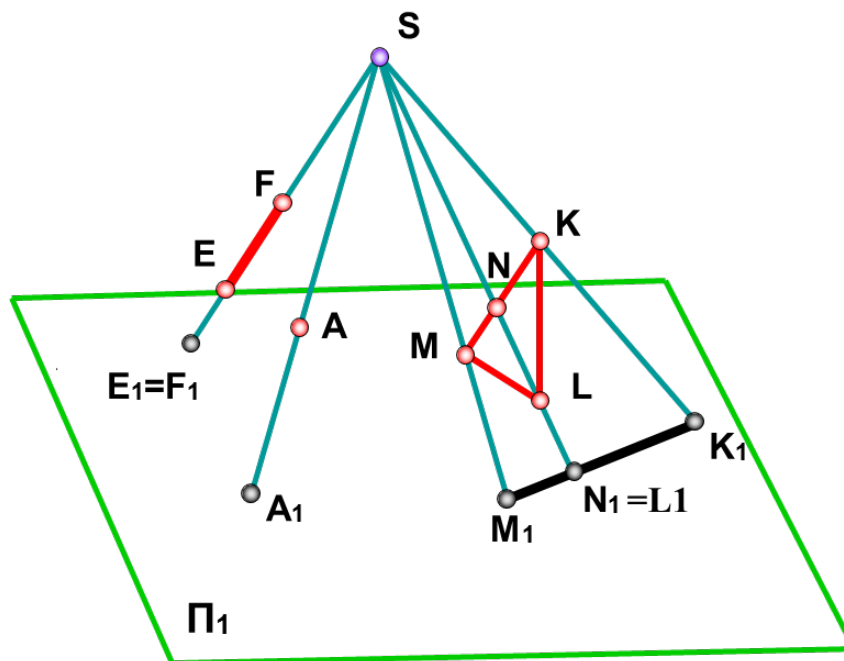


Fig. 1.3

Parallel Projection

When center of projection S is a point at infinity, all projectors (AA_1 , BB_1 , CC_1) are parallel. Such a projection is called parallel projection. In parallel projection center of projection is replaced by vector of projection. So, parallel projection can be considered as a particular case of central projection.

If angle between projectors and a plane of projection is less than 90° , $\alpha \neq 90^\circ$, then it is parallel nonorthogonal projection. If angle between projectors and a plane of projection is 90° (projectors are perpendicular to the picture plane), then it is parallel orthogonal (from Greek right angular) or orthographical projection.

Properties of Parallel Projection

All the properties of central projection are preserved. New properties are added (Fig. 1.4):

1. Parallel lines project to parallel lines ($(CD) \parallel (AB) \rightarrow (C_1D_1) \parallel (A_1B_1)$).

2. Ratio between line segments of the same line or parallel lines preserves

$$\frac{[AN]}{[NB]} = \frac{[A_1N_1]}{[N_1B_1]} \text{ (Fig. 1.4).}$$

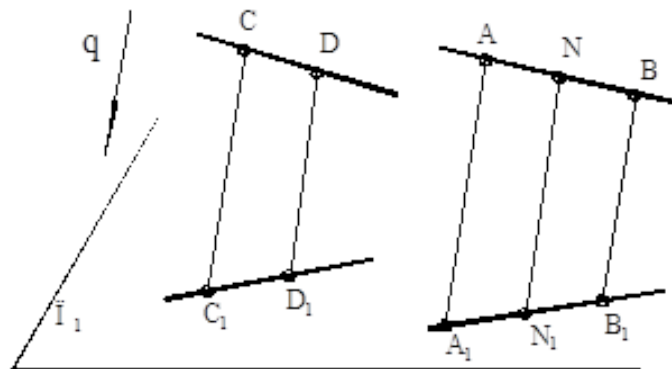


Fig. 1.4

3. Planar figure, parallel to the plane of projection, projects to a true size onto that plane (Fig. 1.5).

4. For orthographical projection sizes of the image can't exceed sizes of the original. If planar object is parallel to the plane of projection, then it projects to a true size.

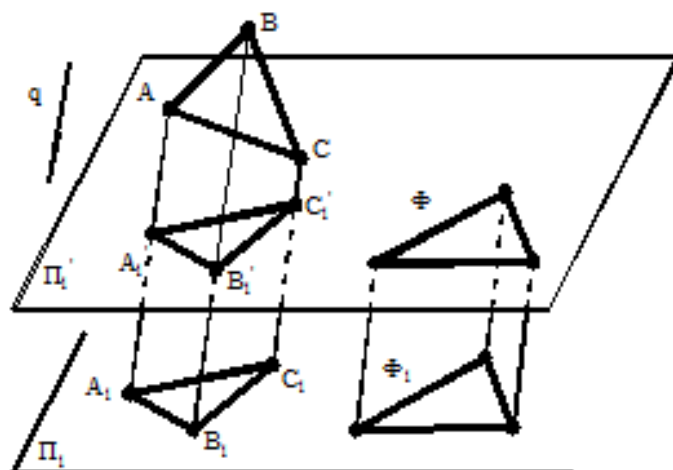


Fig. 1.5

Problems

- 1.1. Parallel projections A_1B_1 and C_1D_1 of the lines AB and CD are parallel lines. Determine possible relative positions of the lines AB and CD .
- 1.2. Parallel projections A_1B_1 and C_1D_1 of the lines AB and CD are intersecting lines. Determine possible relative positions of the lines AB and CD .
- 1.3. $|AB|=40$, and its parallel projection $|A_1B_1|=40$. Determine relative positions between AB and A_1B_1 .
- 1.4. Triangle ABC : $|AB|=30$, $|BC|=40$, $|AC|=50$. Parallel projection of the triangle ABC is $A_1B_1C_1$, where $|A_1B_1|=25$, $|B_1C_1|=50$, $|A_1C_1|=40$. What type of parallel projection is applied orthogonal or nonorthogonal.
- 1.5. $A_1B_1C_1$ is parallel projection of the triangle ABC . $A_1(100,40)$, $B_1(25,15)$ and $C_1(10,55)$. Construct projections of altitudes in the triangle $A_1B_1C_1$.
- 1.6. $A_1(100,40)$, $B_1(70,55)$ and $O_1(55,35)$ are parallel projections of the points of equilateral hexagon $ABCDEF$, where O – center of the hexagon, A and B are vertexes of the hexagon.

2. ORTHOGRAPHIC PROJECTION OF A POINT

Reversible images

Images on the drawings should be reversible to imagine real shape of an object. It's impossible to restore the shape of the figure by only one orthographic projection. In Figure 2.1 projections of the points A, B and C are coinciding in one point $A_1=B_1=C_1$, because points A, B and C are allocated on the same projector. The same situation is with the curves EFD and MNL (see Fig. 2.1).

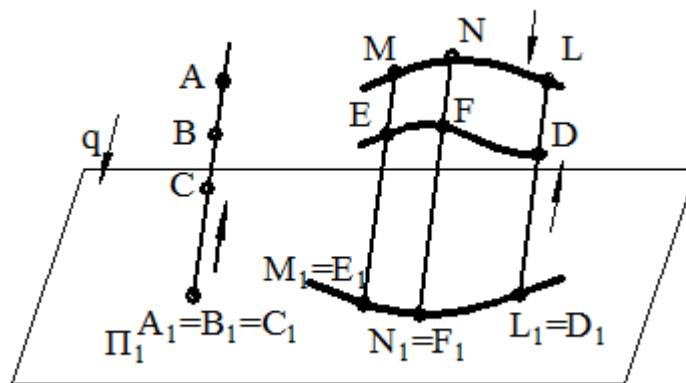


Fig. 2.1

As it goes from Fig. 2.1 it's necessary to provide reversibility of images in the drawings using some methods.

Methods to obtain reversible images:

To obtain reversible images some additional terms have to be met.

To meet these terms, the following methods can be used:

1. Orthographical projection on two planes (Monge's method);
2. Axonometric projection method;
3. Method of projection with elevations;
4. Vector constructions.

There are two ways to provide reversible images using projections without additional constructions (Fig. 2.2):

1. A point projects onto two directions or two different planes.
2. Points of original object are obtained from intersection of corresponding projectors.

In the first case point A projects parallel to two projecting vectors q' and q'' onto the same plane Π_1 , in the second case it projects parallel to projecting vectors q' and q'' onto different planes Π_1 and Π_2 .

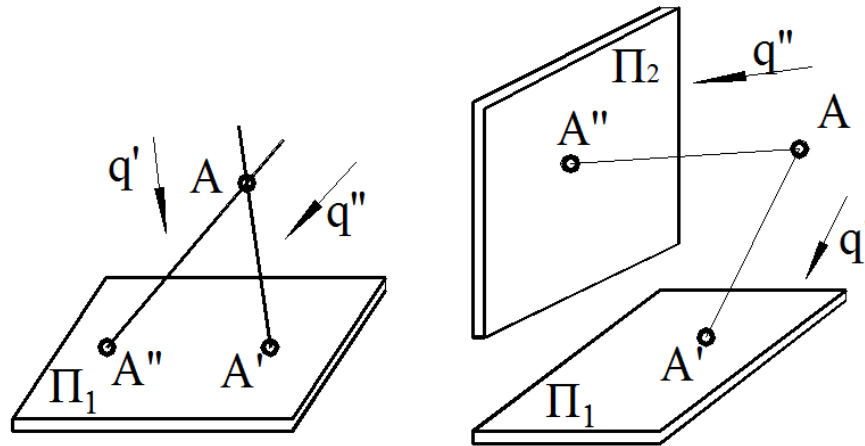


Fig. 2.2

Monge's method

According to Gaspard Monge's method (Fig. 2.3) a point projects onto two mutually perpendicular principal planes (planes of projection). After that, two principal planes are superposed by rotating one plane around the axes (which is a line of intersection between two planes of projection) up to coinciding with the other plane (rotating horizontal principal plane Π_1 around x-axis).

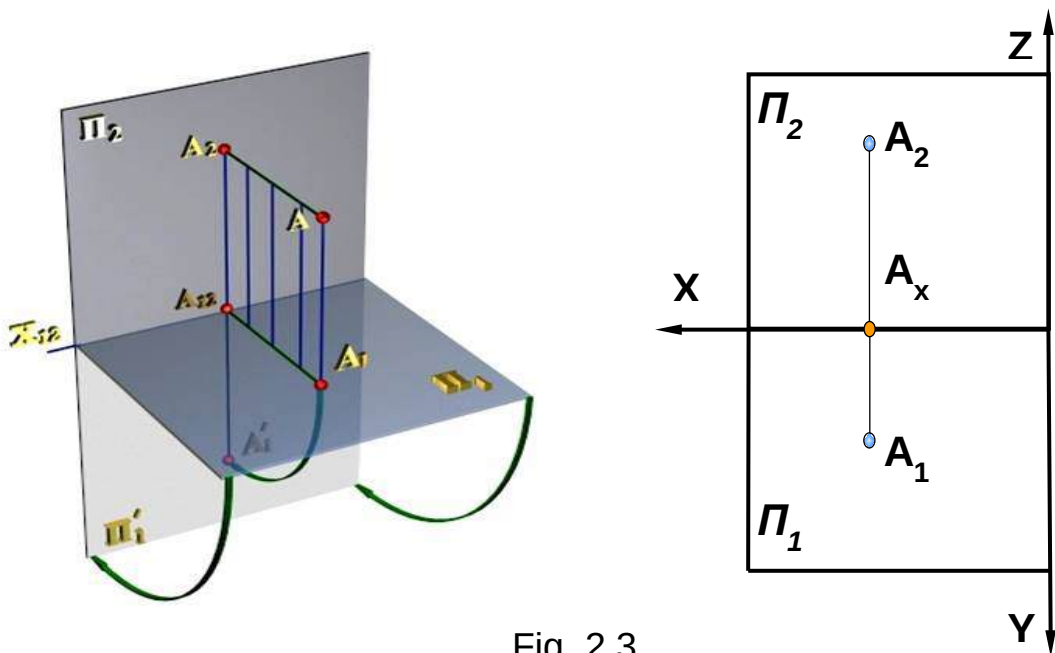


Fig. 2.3

Designations

Π_1 – horizontal principal plane (plane of projection) - H;

Π_2 – frontal (vertical) principal plane (plane of projection - F (V);

A_1 – horizontal projection of a point A;

A_2 – frontal projection of a point A;

$A_x A_1$ – depth of A-point (distance from Π_2 -plane);

$A_x A_2$ – height of A-point (distance from Π_1 -plane);

$A_1 A_2$ – projectors (projecting lines).

Horizontal and frontal projections of a point lie on the same vertical projector, perpendicular to x-axis.

This type of projection avoids the problem when it's impossible to discern location of the points, which have same projections on some planes. These points are called crossing points or concurrent points (Fig. 2.4).

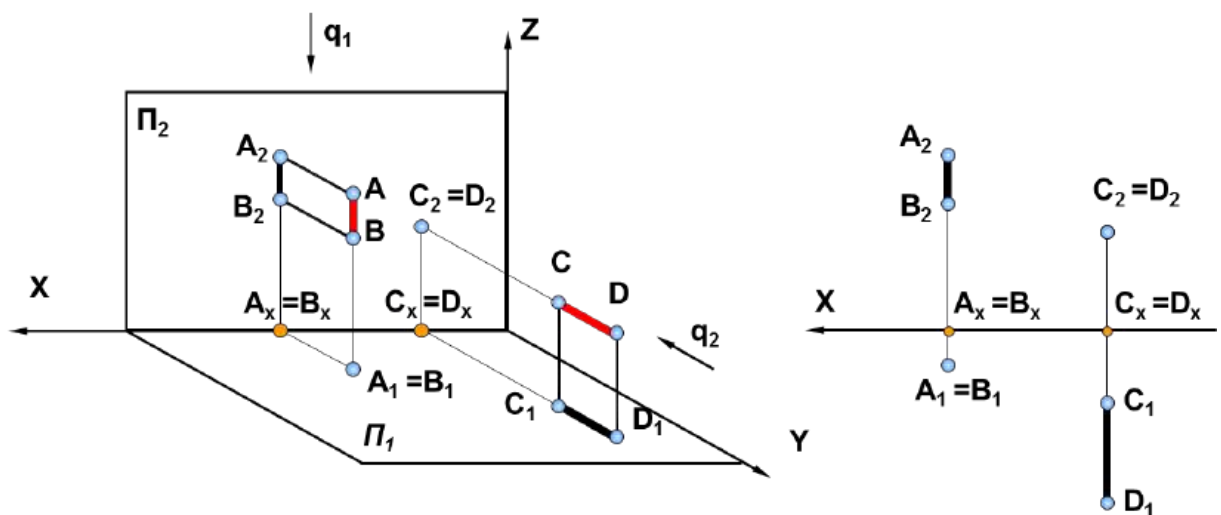


Fig. 2.4

If projections of two different points coincide on one picture plane, they are different on the other picture plane.

For example, projections A_1 and B_1 of the points A and B (see Fig. 2.4) coincide on the plane Π_1 ($A_1=B_1$), but different on the plane Π_2 ($A_2 \neq B_2$), i.e. coordinates x and y of the points A and B are same, while coordinate z is different. For the points C and D projections C_2 and D_2 (see Fig. 2.4) coincide on the plane Π_2 ($C_2=D_2$), but different on the plane Π_1 ($C_1 \neq D_1$), i.e. coordinates x and z of the points C and D are same, while coordinate y is different. So, points A and B are called horizontally concurrent points, points C and D - frontally concurrent points.

Orthographic projection on three planes.

Some problem can be solved easier, if two principle planes of projection would be supplemented by the third plane, perpendicular to them. Such a plane is called profile plane of projection (Π_3).

This principal plane (Fig. 2.5) is superposed by rotating around the axes (which is a line of intersection between two planes of projection) up to coinciding with the combined planes (rotating profile principal plane Π_3 around z-axis).

Coordinates of the point A: X_A , Y_A , Z_A – numeric values of line segments along coordinate axes.

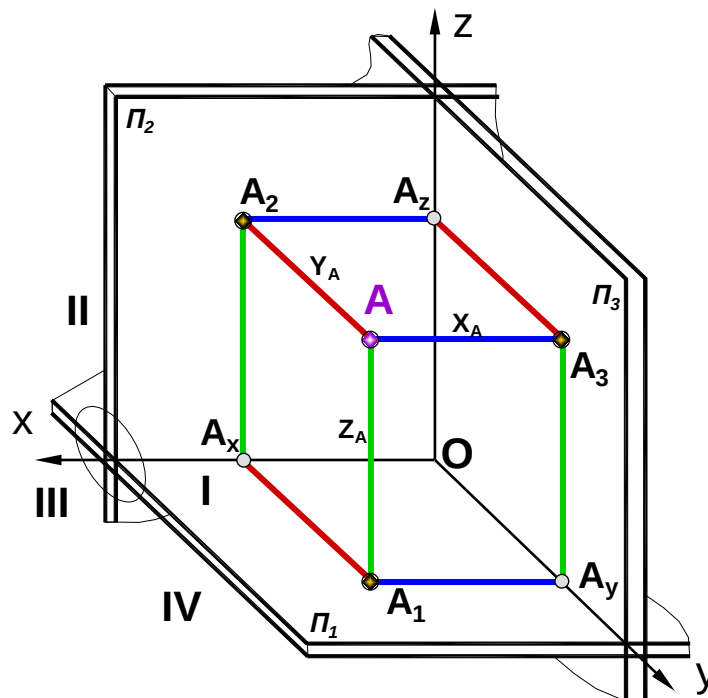


Fig. 2.5

Designations:

- Π_1 – horizontal principal plane (plane of projection) - H;
- Π_2 – frontal (vertical) principal plane (plane of projection - F (V);
- Π_3 – profile (vertical) principal plane (plane of projection - P (W);
- A_1 – horizontal projection of a point A;
- A_2 – frontal projection of a point A;
- A_3 – profile projection of a point A;
- $A_x A_1$ – depth of A-point (distance from Π_2 -plane);
- $A_x A_2$ – height of A-point (distance from Π_1 -plane);
- $A_1 A_2$ – projectors (projecting lines).
- X, Y, Z – axes;

A_1, A_2, A_3 – projections of A-point;

$AA_3=XA; AA_2=YA; AA_1=ZA;$

A_xA_1 – depth;

A_xA_2 – height;

OA_x – width.

For engineering purposes entire space is divided on four quadrants (profile plane is not included into consideration) or eight octants (profile plane is included into consideration). Thus, if an object is located in the first quadrant, such a method is called First angle projection method. If an object is located in the third quadrant, such a method is called Third angle projection method.

Algorithm to construct orthographic projection of a point (Fig. 2.6):

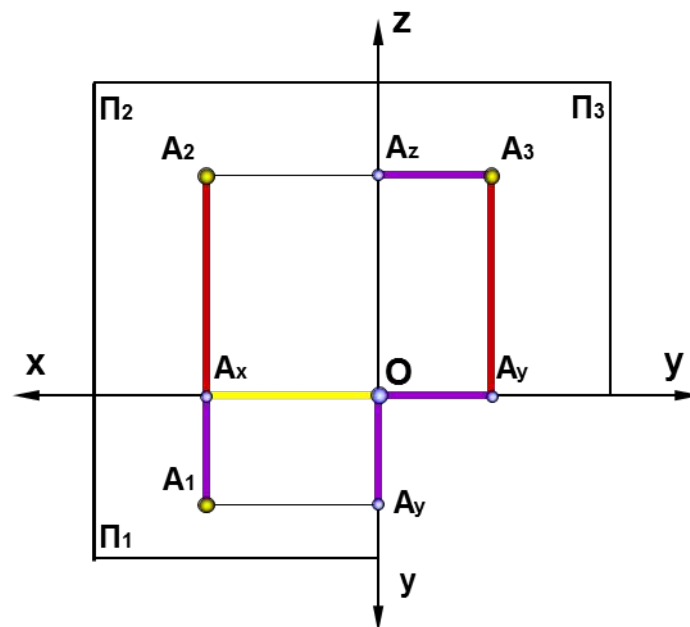


Fig. 2.6

1. Drop X_A coordinate (width) from origin point along X-axis, designated A_x ($OA_x = X_A$).
2. Construct vertical projector $\perp$ X-axis.
3. Construct A_2 protracting Z_A - coordinate (height) along z-axis ($A_xA_2 = Z_A$).
4. Construct A_1 protracting Y_A - coordinate (depth) along y-axis ($A_xA_1 = Y_A$).
5. Construct horizontal projector $\perp$ Z-axis and find A_z .

6. Construct A_3 protracting Y_A - coordinate (depth) along horizontal y -axis ($A_2A_3 = Y_A$).

This algorithm allows constructing orthographic projections of the point A, knowing its coordinates $x_A = OA_x$ – width of the point A, $y_A = OA_y$ – depth of the point A, $z_A = OA_z$ – height of the point A.

Line $A_2A_1A_3$ – vertical projector – connects horizontal and frontal projections of the point A; A_2A_3 – horizontal projector – connects frontal and profile projections of the point A.

Relative position of points on orthographic view. Axes in a drawing

Orthographic projections of the points A and B are shown in Fig. 2.7. Projection of the point B is constructed in the similar way, as projection of the point A. Coordinate $x_B = OB_x$ – width of the point B, $y_B = OB_y$ – depth of the point B, $z_B = OB_z$ – height of the point B.

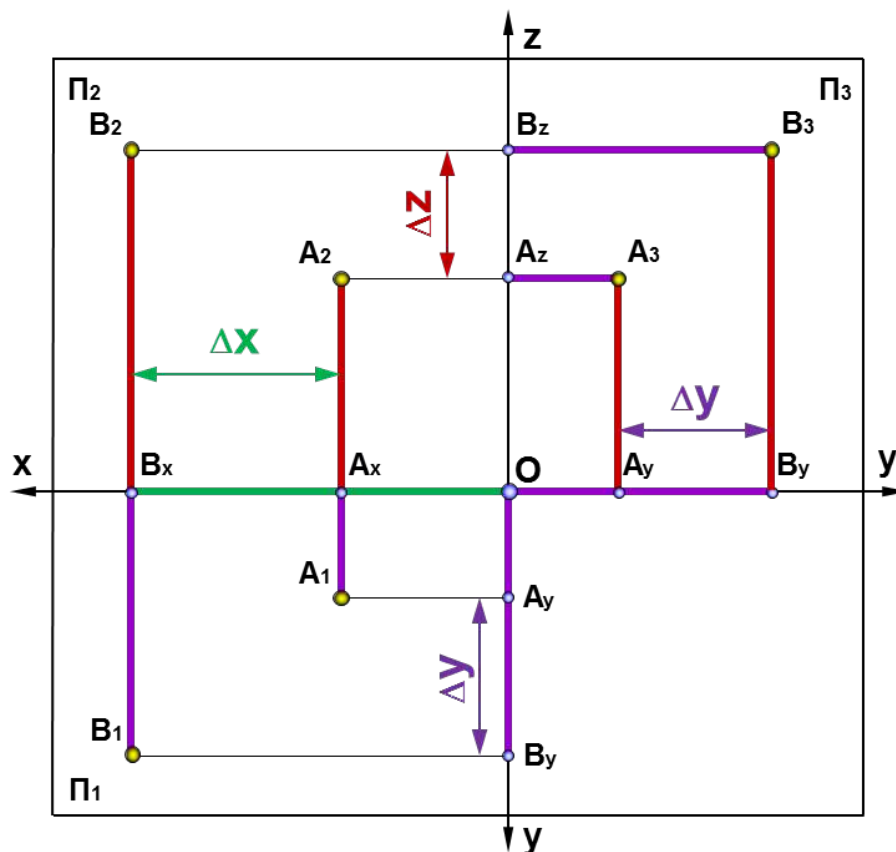


Fig. 2.7

As it is seen from Fig. 2.7 point B is higher than the point A, because z -coordinate of the point B is greater than z -coordinate of the point A, and point B is in front of, because y -coordinate of the point B is greater than y -coordinate of the point A.

Differences in x(width), y(depth) and z(height) coordinates:

- $\Delta x = |OB_x - OA_x|$;
- $\Delta y = |OBy - OAy|$;
- $\Delta z = |OBZ - OAZ|$.

If axes and origin point are not indicated in the drawing, it's impossible to define absolute coordinates of separately taken point.

But it's possible to determine relative coordinates of any point with respect to coordinates of other points, which is more important in engineering graphics.

Orthographic projections of the same points A and B but without indication of coordinate axes in the drawing are shown in Fig. 2.8.

Relative coordinates of the points A and B are preserved.

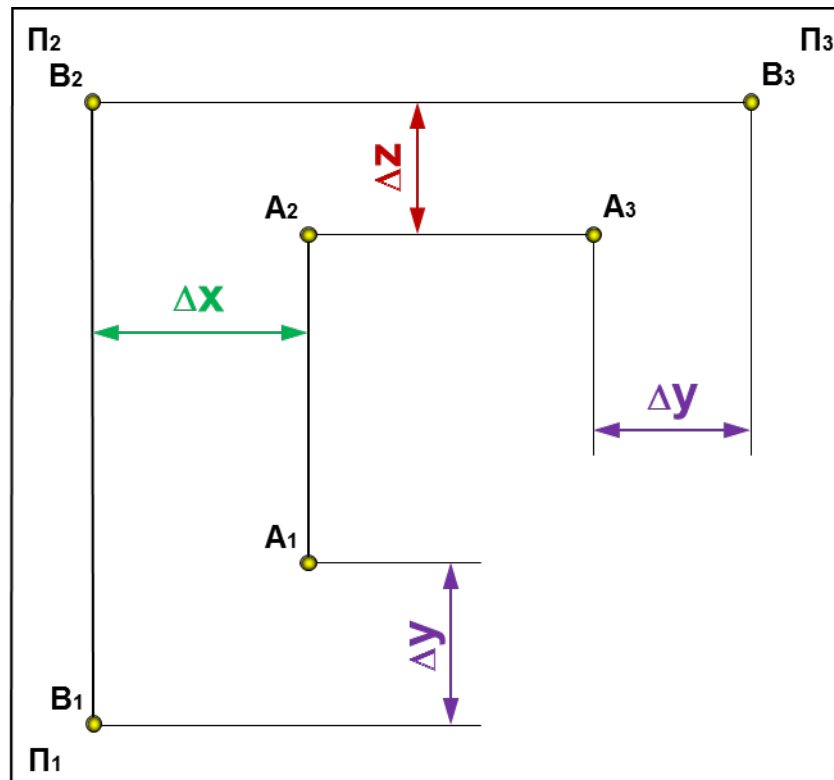


Fig. 2.8

Problems

2.1. Construct three orthographic views for the points: A(60,30,40), B(50,40,0), C(40,0,30), D(0,50,50), E(30,0,0), K(0,20,0), L(0,0,20), M(20,-20,30), N(10,20,-30) and Q(-10,20,30).

- 2.2. $A_1(40,30)$ – horizontal and $A_2(40,20)$ – frontal orthographic projections of the point A. Construct left side view A_3 - orthographic projection of the point A.
- 2.3. $A_1(40,30)$ – horizontal and $A_3(30,20)$ – profile orthographic projections of the point A. Construct frontal orthographic projection A_2 - front view of the point A.
- 2.4. $A_2(30,20)$ – frontal and $A_3(40,20)$ – profile orthographic projections of the point A. Construct horizontal orthographic projection A_1 - horizontal view of the point A.
- 2.5. Point A(30,40,20). Construct three views of the point B, symmetric to the point A with respect to XOY - horizontal principal plane.
- 2.6. Point A(30,40,20). Construct three views of the point B, symmetric to the point A with respect to XOZ - frontal principal plane.
- 2.7. Point A(30,40,20). Construct three views of the point B, symmetric to the point A with respect to YOZ - profile principal plane.
- 2.8. Point A(30,40,20). Construct three views of the point B, symmetric to the point A with respect to OX-axis.
- 2.9. Point A(30,40,20). Construct three views of the point B, symmetric to the point A with respect to OY-axis.
- 2.10. Point A(30,40,20). Construct three views of the point B, symmetric to the point A with respect to OZ-axis.
- 2.11. Point A(30,40,20). Construct three views of the point B, symmetric to the point A with respect to the origin point O.

3. ORTHOGRAPHIC PROJECTION OF A STRAIGHT LINE

Straight Line. Representation in a drawing

If we join proper projections of the points A and B (see Fig. 2.7, 2.8), we obtain horizontal, frontal and profile orthographic projections of the line segment AB (Fig. 3.1).

Each coordinate of AB line segment can be determined from the drawing.

Thus, we can say that AB line segment is represented in orthographic drawing.

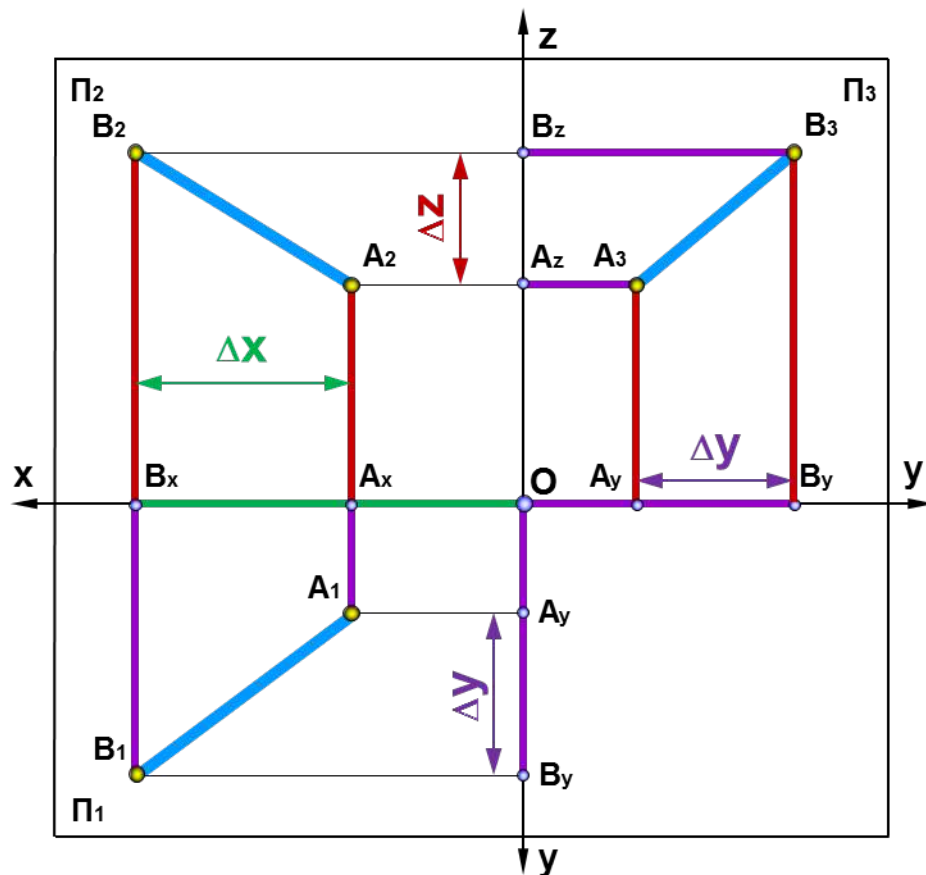


Fig. 3.1

Line segment can be represented in the orthographic drawing even if axes and origin point are not indicated (Fig. 3.2).

It's impossible to define absolute coordinates of each separately taken point, but it's possible to determine relative coordinates of each point of the line segment with respect to coordinates of other points of the line AB.

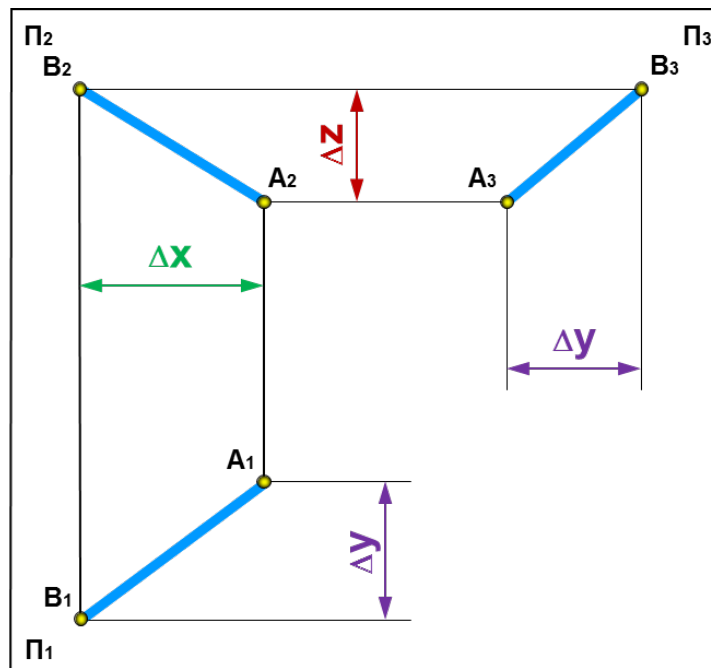


Fig. 3.2

Straight Lines. Classification. Principal Lines

Lines can be classified with respect to principal planes of projection. Some lines are located parallel or perpendicular to the principal planes of projection. These lines are called principal lines. If a line is neither parallel nor perpendicular to any principal plane, such a line is called Oblique line. Classification of lines is given in Fig. 3.3.

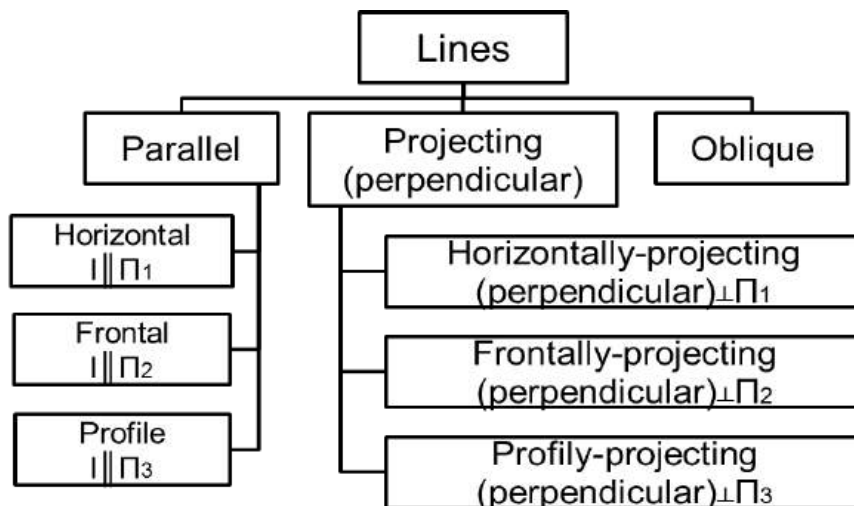


Fig. 3.3

Oblique line

Oblique Line is neither parallel nor perpendicular to any principal plane of projection (see Fig. 3.1, 3.2).

Oblique line projects onto principal planes Π_1 , Π_2 and Π_3 to line segments of a length, less than a true size.

Lengths of the projections can be found through angles between a line segment and principal planes of projection (Π_1 , Π_2 and Π_3 , or H, V and W):

α - angle between a line segment and horizontal principal plane (Π_1 or H);

β - angle between a line segment and frontal principal plane (Π_2 or V);

γ - angle between a line segment and profile principal plane (Π_3 or W).

$$|A_1B_1|=|AB|\cdot\cos\alpha$$

$$|A_2B_2|=|AB|\cdot\cos\beta \quad (3.1)$$

$$|A_3B_3|=|AB|\cdot\cos\gamma$$

Horizontal line

Horizontal (horizontally parallel) line is a straight line, parallel to horizontal principal plane Π_1 (Fig. 3.4).

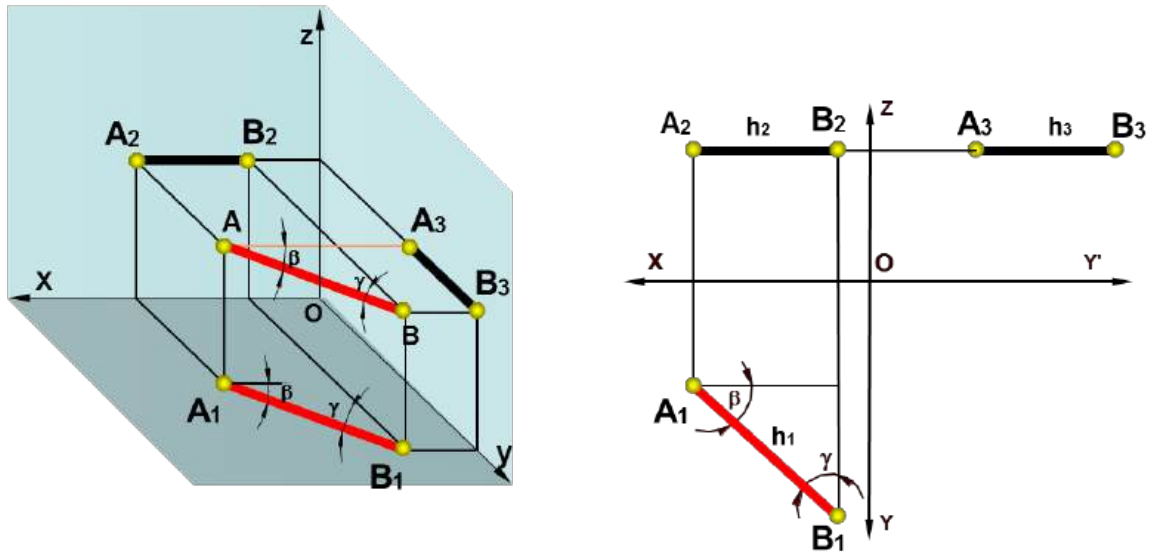


Fig. 3.4

Horizontal line projects onto Π_1 to a True Length.

Angles β and γ on that projection are true angles between a line and principal planes of projection Π_2 and Π_3 .

Horizontal line projects onto Π_2 and Π_3 to horizontal line segments of a length, less than a true size (3.1).

Frontal line

Frontal (Frontally parallel) line is a straight line, parallel to horizontal principal plane Π_2 (Fig. 3.5).

Frontal line projects onto Π_2 to a True Length.

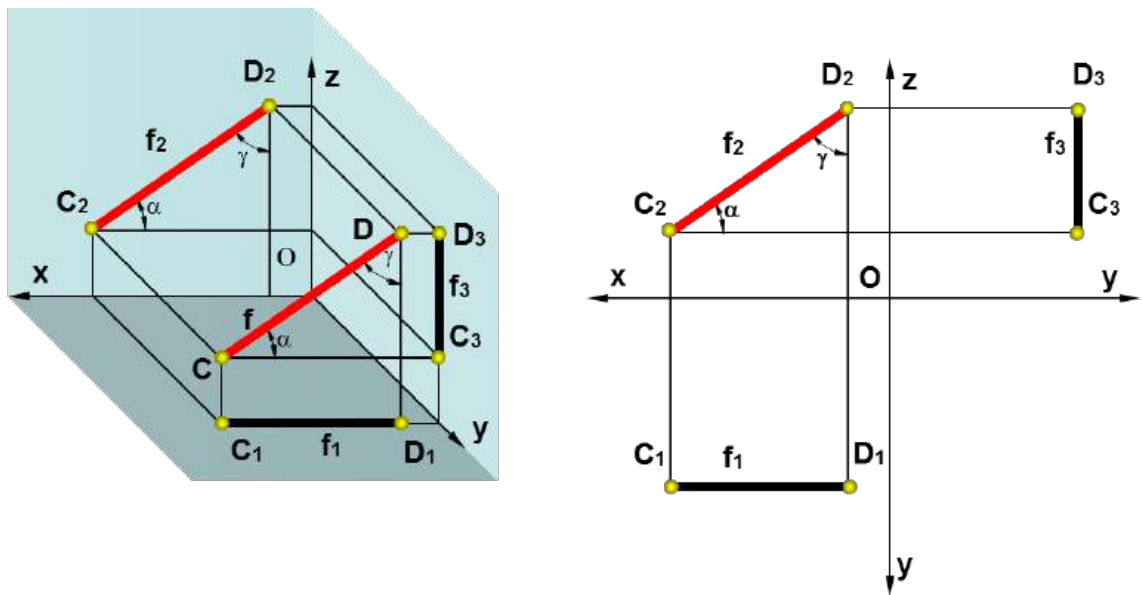


Fig. 3.5

Angles α and γ on that projection are true angles between a line and principal planes of projection Π_1 and Π_3 .

Frontal line projects onto Π_1 and Π_3 to vertical line segments of a length, less than a true size (3.1).

Profile line

Profile (Profile parallel) line is a straight line, parallel to profile principal plane Π_3 (Fig. 3.6).

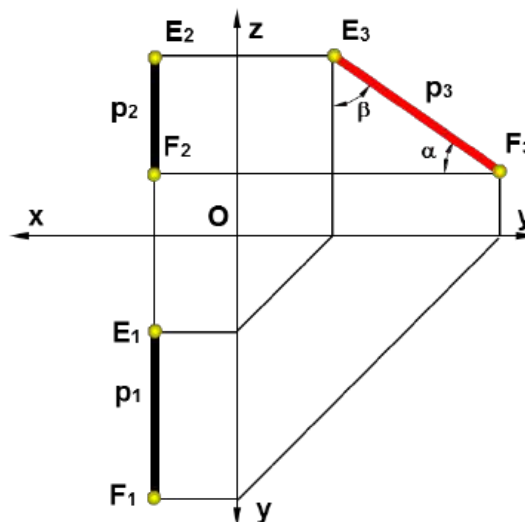


Fig. 3.6

Profile line projects onto Π_3 to a True Length. Angles α and β on that projection are true angles between a line and principal planes of projection Π_1 and Π_2 . Profile line projects onto Π_1 and Π_2 to vertical line segments of a length, less than a true size.

Horizontally-projecting line

Horizontally-projecting (horizontally perpendicular) line is a straight line, perpendicular to horizontal principal plane Π_1 (Fig. 3.7).

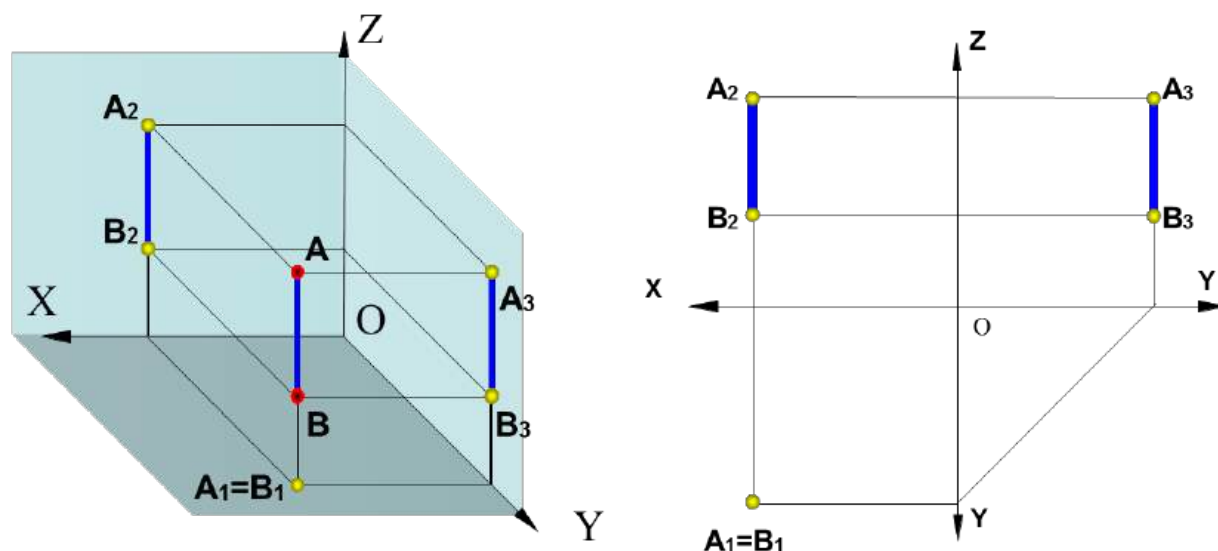


Fig. 3.7

Line segment projects to a point onto Π_1 .

Projections on Π_2 and Π_3 are vertical lines of a true length.

Frontally-projecting and Profile-projecting lines have similar properties.

Problems

- 3.1. Through the point A(35,25,15) draw horizontal line h at an angle of 60° to the frontal principal plane.
- 3.2. On the orthographic drawing determine distance between points A(50,40,20) and B(10,10,20).
- 3.3. Through the point A(40,15,25) draw frontal line f at an angle of 30° to the horizontal principal plane.
- 3.4. On the orthographic drawing determine distance between points A(50,20,40) and B(10,20,10).
- 3.5. Through the point A(25,30,35) draw profile line p at an angle of 60° to the frontal principal plane.
- 3.6. Through the point A(20,20,10) draw horizontally-perpendicular line segment of the length 25 mm.
- 3.7. Through the point A(20,10,20) draw frontally-perpendicular line segment of the length 25 mm.
- 3.8. Through the point A(10,20,20) draw profile-perpendicular line segment of the length 25 mm.

4. RELATIVE POSITION OF A POINT AND A LINE

A point can belong to a line or does not belong to a line. From the property of a projection it is known: if a point belongs to a line, then projection of that point belongs to the projection of that line (Fig. 4.1).

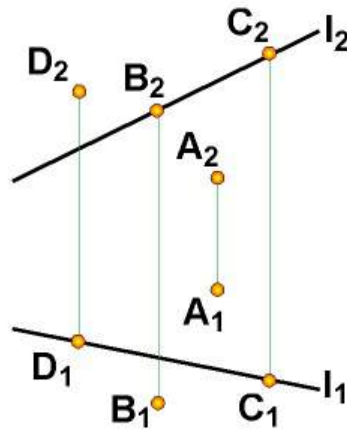


Fig. 4.1

If a point does not belong to a line, then with respect to the principal planes of projection a point can be: above, below, in front of, behind (see Fig. 4.1).

Problems

4.1. Points $A(90,40,40)$ and $B(0,20,20)$. Construct orthographic drawings of the points:

- C, allocated in front of the line AB, if distance between the point C and frontal principal plane is 80 mm;
- D, allocated behind the line AB, if distance between the point D and profile principal plane is 70 mm;
- E, allocated on the line AB, if distance between the point E and profile principal plane is 60 mm;
- K, allocated above the line AB, if distance between the point K and profile principal plane is 50 mm;
- L, allocated below the line AB, if distance between the point L and profile principal plane is 40 mm;
- M, allocated below and in front of the line AB, if distance between the point M and frontal principal plane is 30 mm;
- N, allocated above and behind the line AB, if distance between the point N and frontal principal plane is 20 mm.

4.2. Points A(70,50,30) and B(10,10,10). Construct orthographic drawing of the line segment AB and points C and D, which belong to the line AB, if:

- distance between the point C and horizontal principal plane is 25 mm;
- distance between the point D and frontal principal plane is 20 mm.

4.3. Points A(60,50,10) and B(10,20,40). On the AB-line segment find the point, equally spaced from horizontal and frontal principal planes.

4.4. Points A(60,20,10) and B(10,30,50). On the AB-line segment find the point, equally spaced from horizontal and profile principal planes.

4.5. Points A(60,10,10) and B(10,30,40). On the AB-line segment find the point, equally spaced from frontal and profile principal planes.

4.6. Points A(60,40,30) and B(10,10,20). Draw orthographic drawing of AB-line segment and the point C, which divides line segment AB at the ratio 2:5.

4.7. Points A(50,40,20) and B(10,10,20). Draw orthographic drawing of the line segment AB and the point C, which belongs to the line AB and spaced from the point A at the distance of 20 mm.

4.8. Points A(50,20,40) and B(10,20,10). Draw orthographic drawing of the line segment AB and the point C, which belongs to the line AB and spaced from the point A at the distance of 20 mm.

4.9. Points A(60,20,40) and B(60,40,10). Draw orthographic drawing of the line segment AB and the point C, which belongs to the line AB and spaced from the point A at the distance of 20 mm.

4.10. Points A(30,20,10), B(30,20,40) and C(15,10,20). Draw orthographic drawing of the line segment AB, points C and D, if point D is symmetric to the point C with respect to the line AB.

5. RELATIVE POSITION OF LINES

Two straight lines in a space can be coinciding, intersecting, parallel or nonintersecting (skew or crossing) with respect each to other (Fig. 5.1).

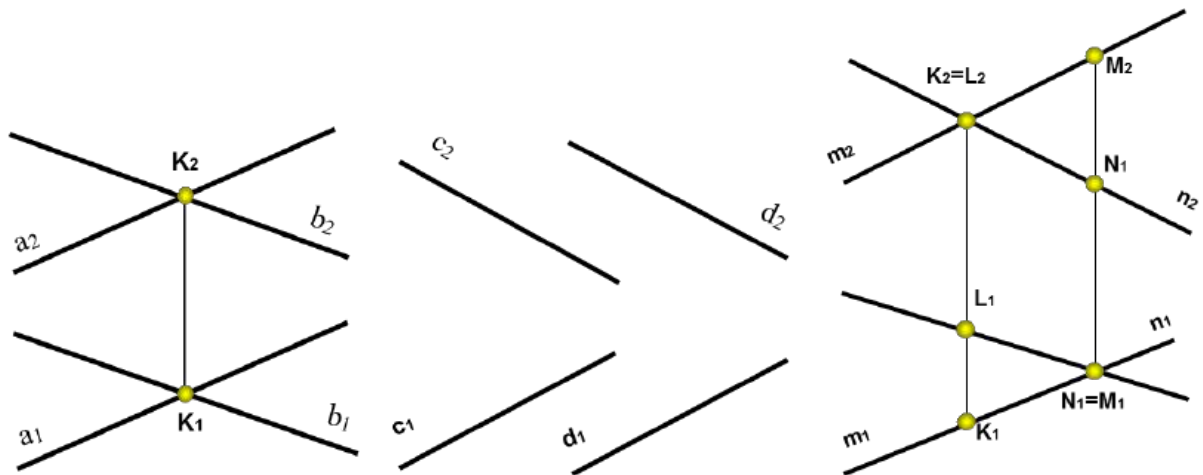


Fig. 5.1

Intersecting lines a and b have real point of intersection, which projections K_1 and K_2 are located on the same projector.

Parallel lines c and d have proper parallel projections and intersect in a point at infinity.

Skew lines m and n don't have common point and don't form a plane. Points K and L , M and N are called concurrent (crossing) points.

Problems

- 5.1. Points $A(50,20,20)$, $B(10, _, 50)$, $C(30,10,10)$ and $D(10,20, _)$. Complete orthographic projections of the parallel lines AB and CD .
- 5.2. Points $A(55,10,35)$, $B(10,30,40)$, $C(50,40,15)$ and $D(15,10, _)$. Construct orthographic projections of skew (crossing) lines AB and CD . Indicate horizontally crossing points.
- 5.3. Points $A(55,10,35)$, $B(10,30,40)$, $C(50,40,15)$ and $D(15,10, _)$. Construct orthographic projections of intersecting lines AB and CD .
- 5.4. Points $A(55,10,10)$, $B(10,30,40)$, $C(50, _, 35)$ and $D(15,10,15)$. Construct orthographic projections of skew (crossing) lines AB and CD . Indicate frontally crossing points.
- 5.5. Points $A(55,10,10)$, $B(10,30,40)$, $C(50, _, 35)$ and $D(15,10,15)$. Construct orthographic projections of intersecting lines AB and CD .

- 5.6. Points A(50,20,10), B(10,5,40) and C(20,20,15). On the orthographic drawing through the point C draw a straight line, which intersects line AB and Y-axis.
- 5.7. Points A(50,10,10), B(10,30,35) and C(20,10,15). On the orthographic drawing through the point C draw a straight line, which intersects line AB and Z-axis.
- 5.8. Points A(15,45,10), B(50,40,25) and C(10,10,25). Construct orthographic drawing of the line segments AB and CD, if AB is perpendicular to CD and point D belongs to the line AB.
- 5.9. Points A(15,45,10), B(50,40,25) and C(10,10,25). Construct orthographic drawing and find distance between point A and line BC.
- 5.10. Points A(45,25,40), B(50,10,10) and C(10,10,35). Construct orthographic drawing and find distance between point A and line BC.
- 5.11. Points A(50,35,25), B(10,10,25) and C(15,35,__). Construct orthographic drawing of the line segment AB and projections of the point C, which is spaced from the line AB at a distance of 25 mm.
- 5.12. Points A(50,35,25), B(10,10,25) and C(15,__,45). Construct orthographic drawing of the line segment AB and projections of the point C, which is spaced from the line AB at a distance of 30 mm.

6. ORTHOGRAPHIC PROJECTIONS OF A PLANE

A plane. Representation in a drawing

Lines and points are usually used to represent a plane in orthographic drawing. A plane can be represented by three points which don't form a straight line, by a line and a point which doesn't belong to the line, by two intersecting or two parallel lines, or by a plane segment (Fig. 6.1).

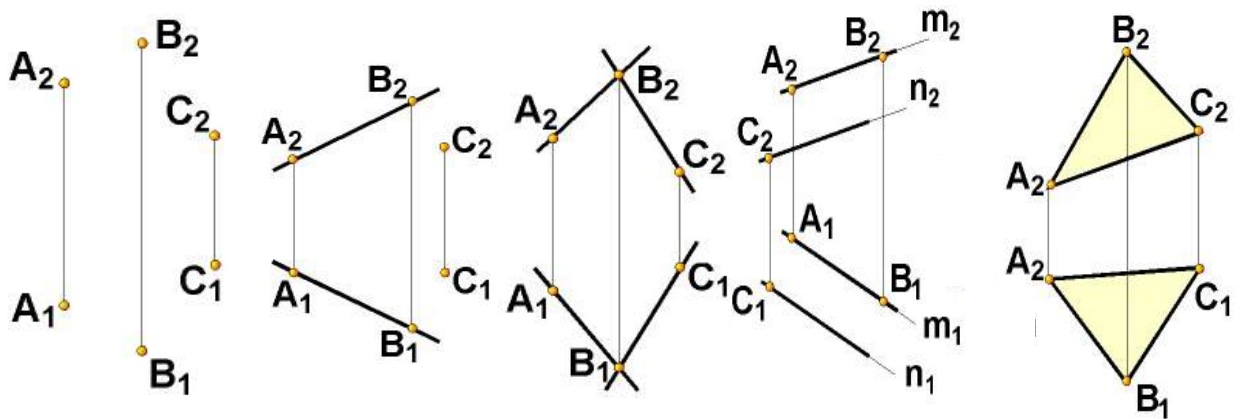


Fig. 6.1

Planes can be classified with respect to principal planes of projection. Some planes are located parallel or perpendicular to the principal planes of projection. These planes are called principal planes. If a plane is neither parallel nor perpendicular to any principal plane, such a line is called Oblique plane. Classification of planes is given in Fig. 6.2.

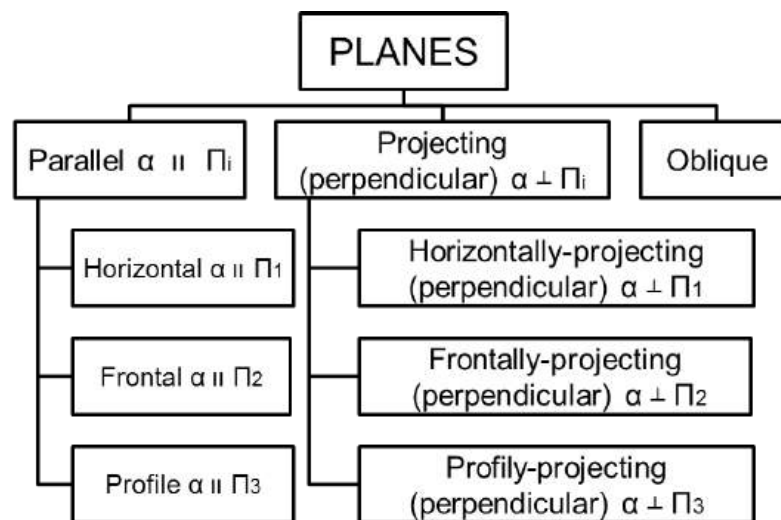


Fig. 6.2

Oblique plane

Oblique plane is neither parallel nor perpendicular to any principal plane of projection (Fig. 6.3).

Oblique plane projects onto principal planes Π_1 , Π_2 and Π_3 to plane segments of an area, less than a true size.

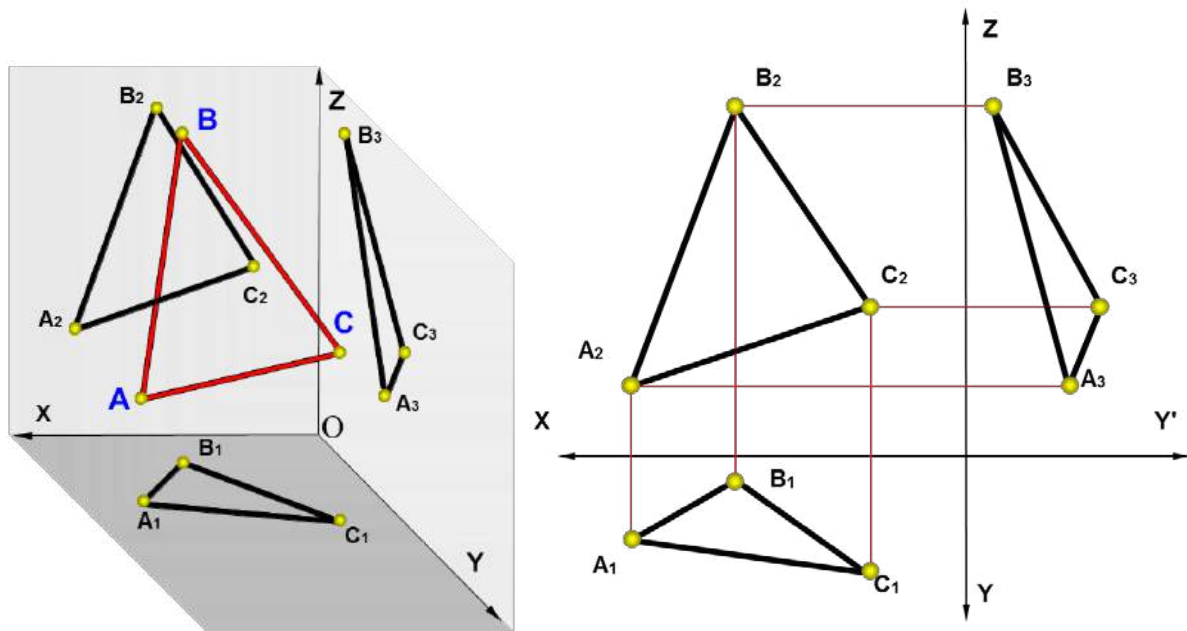


Fig. 6.3

Horizontal plane

Horizontal (principal) plane ((ABC) $\parallel$ Π_1) is a plane, parallel to horizontal principal plane of projection (Fig. 6.4).

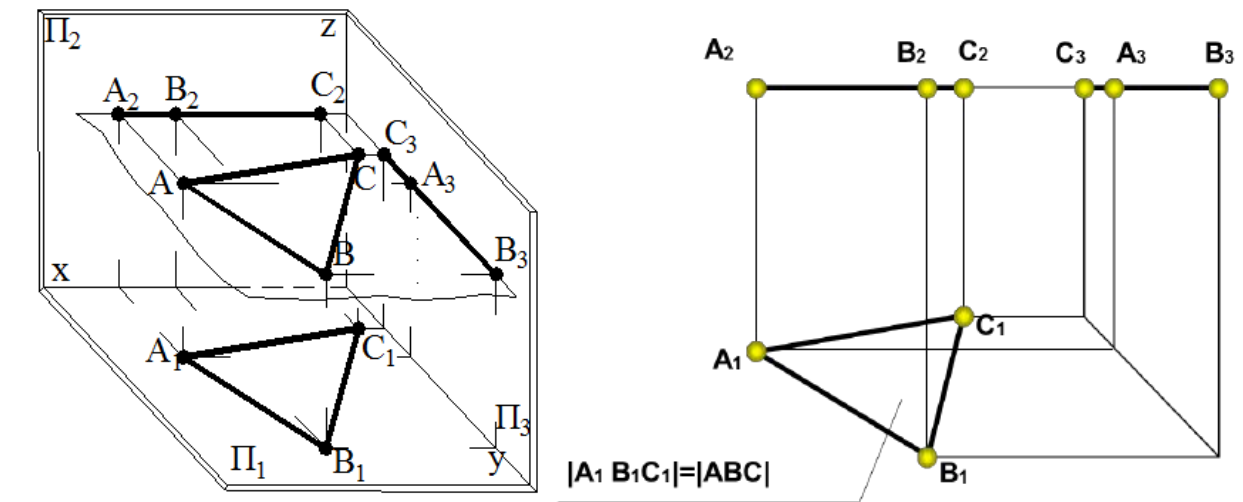


Fig. 6.4

Any planar figure, which lies in this plane, projects onto the horizontal principal plane Π_1 to a true size.

Frontal and Profile planes have similar properties.

Frontally-projecting plane

Frontally-projecting (frontally-perpendicular) plane $((ABC) \perp \Pi_2)$ is a plane, perpendicular to the frontal principal plane of projection (Fig. 6.5). Any planar figure, which lies in this plane, projects onto the frontal principal plane to a straight line. Frontally-projecting plane can be represented by its one frontal projection Σ_2 only. Angles α and γ between Σ_2 and orthogonal projectors are true angles between the plane Σ and principal planes Π_1 and Π_3 .

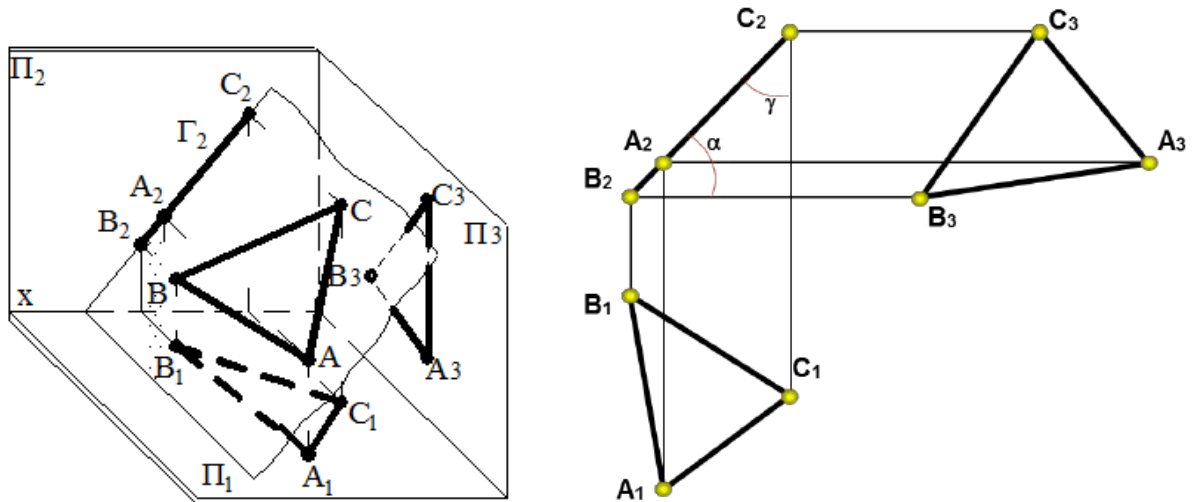


Fig. 6.5

Horizontally-projecting and Profile-projecting planes have similar properties.

Principal Lines in a Plane

Principal Lines in a Plane – lines, which belong to that plane and parallel to the principal planes of projection: **Horizontal h** – line, parallel to Π_1 ; **Frontal f** – line, parallel to Π_2 ; **Profile p** – line, parallel to Π_3 (Fig. 6.6).

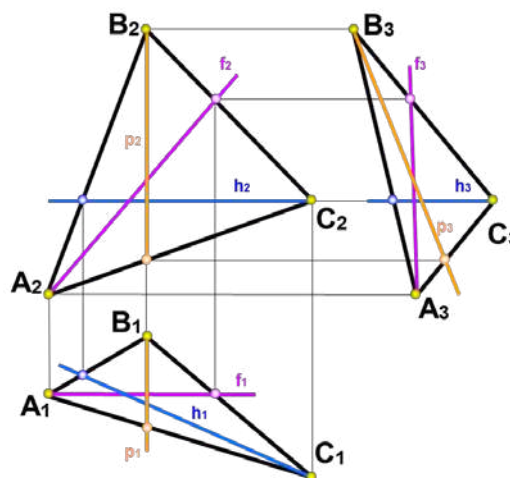


Fig. 6.6

Principal lines in a plane, parallel to one plane of projection, are parallel to each other.

To construct horizontal principal line of ABC-plane first draw its frontal projection, parallel to x-axis, and then draw its frontal and (if necessary) profile projections.

Make similar constructions to obtain frontal or profile principal lines in ABC-plane.

Problems

6.1. Points A(50,10,20) and B(50,40,20). Complete projections of horizontal plane ABC, if point C is equally spaced from frontal and profile principal planes at a distance of 35 mm.

6.2. Points A(50,20,40) and B(10,20,10). Complete projections of frontal plane ABC, if point C is equally spaced from horizontal and profile principal planes at a distance of 35 mm.

6.3. Points A(50,10,10) and B(10,30,40). Complete projections of horizontally perpendicular plane ABC, if point C is equally spaced from horizontal and profile principal planes at a distance of 35 mm.

6.4. Points A(50,10,10) and B(10,30,40). Complete projections of frontally perpendicular plane ABC, if point C is equally spaced from frontal and profile principal planes at a distance of 35 mm.

6.5. Points A(50,10,10) and B(10,30,40). Complete projections of horizontally perpendicular plane ABC, if point C is equally spaced from horizontal and profile principal planes at a distance of 35 mm.

6.6. Points A(60,10,30), B(30,40,50) and C(10,25,10). Draw horizontal line, which belongs to the plane ABC and passes through the point A.

6.7. Points A(60,10,30), B(30,40,50) and C(10,25,10). Draw frontal line, which belongs to the plane ABC and passes through the point C.

6.8. Points A(60,20,20), B(25,10,5), C(10,30,55) and D(45,55,40). Verify, if points A, B, C and D belong to the one plane, or not. Prove it.

6.9. Points A(75,30,25), B(65,65,50), C(25,__,45), D(10,__,30) and E(40,10,5). Complete orthographic drawing of the plane polygon ABCDE.

6.10. Points A(65,40,15), B(35,55,40), C(20,10,5), K(45,__,20) and M(5,20,__). Complete orthographic drawing of straight line segment KM, which lies in the plane ABC.

6.11. Points $A(90, _, 10)$, $B(50, _, 50)$, $C(65, _, 5)$, $K(35, 20, 40)$, $L(5, 5, 45)$, $M(40, 35, 25)$ and $N(10, 20, 30)$. Complete orthographic drawing of the triangle ABC, which lies in the plane, represented by parallel lines KL and NM.

6.12. Points $A(90, 65, 35)$, $B(65, _, 15)$, $C(25, _, 45)$, $D(50, 10, 5)$, $E(10, 35, 35)$ and $K(20, 5, _)$. Complete orthographic drawing of straight line segments AK and BC, if lines AK, BC and DE are located in one plane.

7. RELATIVE POSITION OF A LINE AND A PLANE

There are three possible relative positions between a line and a plane:

- a line lies in a plane;
- a line is parallel to a plane;
- a line intersects a plane.

If two points of a line belong to a plane, then entire line belongs to that plane.

A Line, parallel to a Plane, is a line, which doesn't belong to the plane and doesn't intersect it (or intersects it at infinite point). A line is parallel to a plane, if it is parallel to at least one line, which lies in that plane (Fig. 7.1).

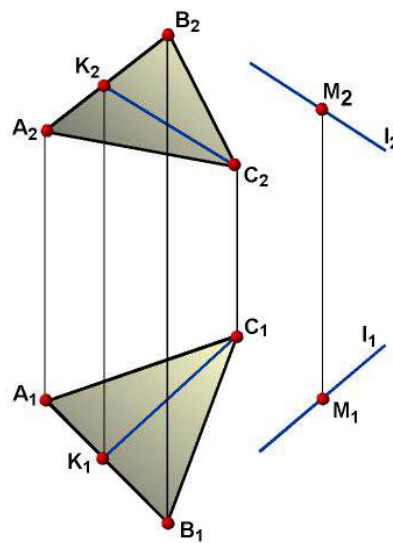


Fig. 7.1

If a line intersects a plane, then a line and a plane have one common point. This point is called piercing point (Fig. 7.2, 7.3).

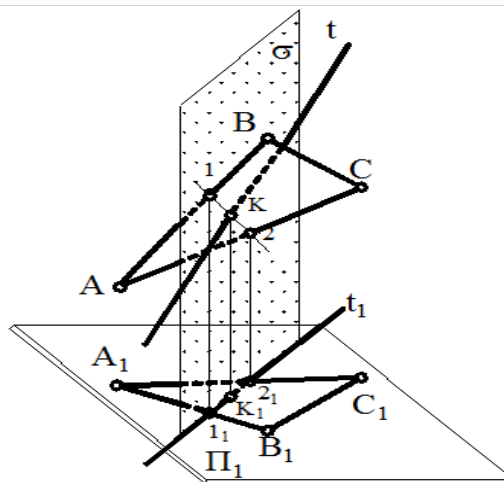


Fig. 7.2

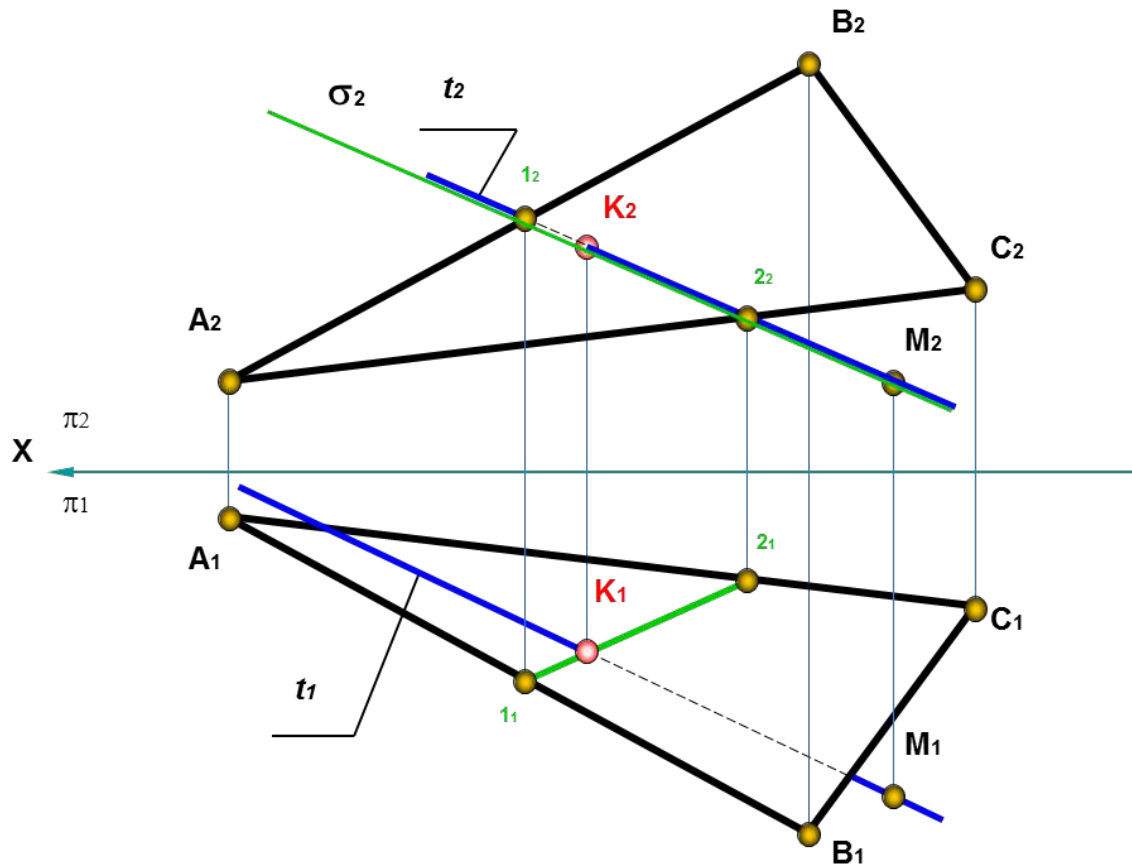


Fig. 7.3

Algorithm to find piercing point:

1. Construct cutting plane which would contain the given line.
2. Find line of intersection between the given plane and cutting plane.
3. Determine point of intersection between obtained line of intersection and given line.
4. Determine visibility of the line segments.

Problems

7.1. Points $A(90,25,25)$, $B(40,45,45)$, $C(60,10,5)$, $K(30,30,15)$ and $M(10,25,40)$. Determine, if straight line KM is parallel to the plane ABC or not.

7.2. Points $A(90,25,25)$, $B(40,45,45)$, $C(60,10,5)$, $K(30, _, 10)$ and $M(10, _, 35)$. Complete orthographic drawing of the straight line KM , if line KM is parallel to the plane ABC .

7.3. Points $A(70,20,25)$, $B(50,10,10)$, $C(35,10,15)$, $D(10,30,25)$ and $K(40,25,30)$. Through the point K draw a plane, which would be parallel to the straight lines AB and CD .

- 7.4. Points $A(55,10,35)$, $B(40,45,25)$, $C(10,15,5)$, $K(60,30,5)$ and $M(5,20,35)$. Construct point of intersection (piercing point) of straight line KM and frontally perpendicular plane ABC. Determine visibility of KM line segment.
- 7.5. Points $A(65,10,20)$, $B(45,30,35)$, $C(20,5,5)$, $K(10,10,30)$ and $M(10,50,30)$. Construct point of intersection (piercing point) between frontally perpendicular line KM and plane ABC. Determine visibility of KM line segment.
- 7.6. Points $A(60,5,20)$, $B(30,40,40)$, $C(10,15,5)$, $K(55,20,10)$ and $M(5,25,25)$. Construct point of intersection (piercing point) between the line KM and the plane ABC. Determine visibility of KM line segment.
- 7.7. Points $A(70,10,20)$, $B(25,20,40)$, $C(50,30,10)$, $D(5,40,30)$, $K(60,45,35)$ and $M(15,5,15)$. Construct point of intersection (piercing point) between the line KM and plane ABCD, represented by the parallel lines AB and CD. Determine visibility of KM line segment.
- 7.8. Points $A(65,40,25)$, $B(40,5,45)$, $C(15,15,5)$ and $P(10,50,50)$. Through the point P draw a straight line, perpendicular to the plane ABC.
- 7.9. Points $A(60,35,35)$, $B(30,15,15)$ and $C(25,40,30)$. Through the point C draw a plane, perpendicular to the line AB.
- 7.10. Points $A(80,15,15)$ and $B(20,75,75)$. Draw locus of points, equally spaced from the points A and B.

8. RELATIVE POSITION OF PLANES

There are three possible relative positions between planes:

- they are coincident;
- they are parallel;
- they are intersecting.

If three points of a plane, which don't form a line, belong to the other plane, then these planes are coincident.

Two planes are parallel, if two intersecting lines of one plane are parallel to two intersecting lines of the other plane.

Line of intersection between two planes is a multiple of points which simultaneously belong to both planes. To find a line of intersection between two planes it's sufficient to find two points which belong to both planes (Fig. 8.1).

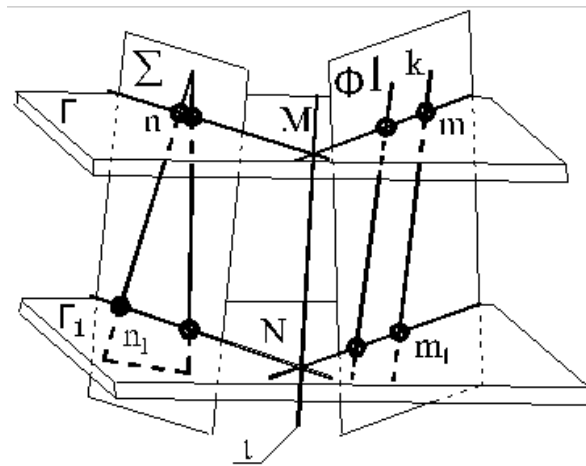


Fig. 8.1

Generally two cutting planes are used to find two common points (Fig. 8.2).

Algorithm to construct line of intersection between two planes:

1. Construct two cutting planes intersecting two given oblique planes and perpendicular to some principal plane.
2. Construct lines of intersection between two cutting planes and two oblique planes.
3. Determine points of intersection between obtained lines.
4. Construct line of intersection joining determined points.

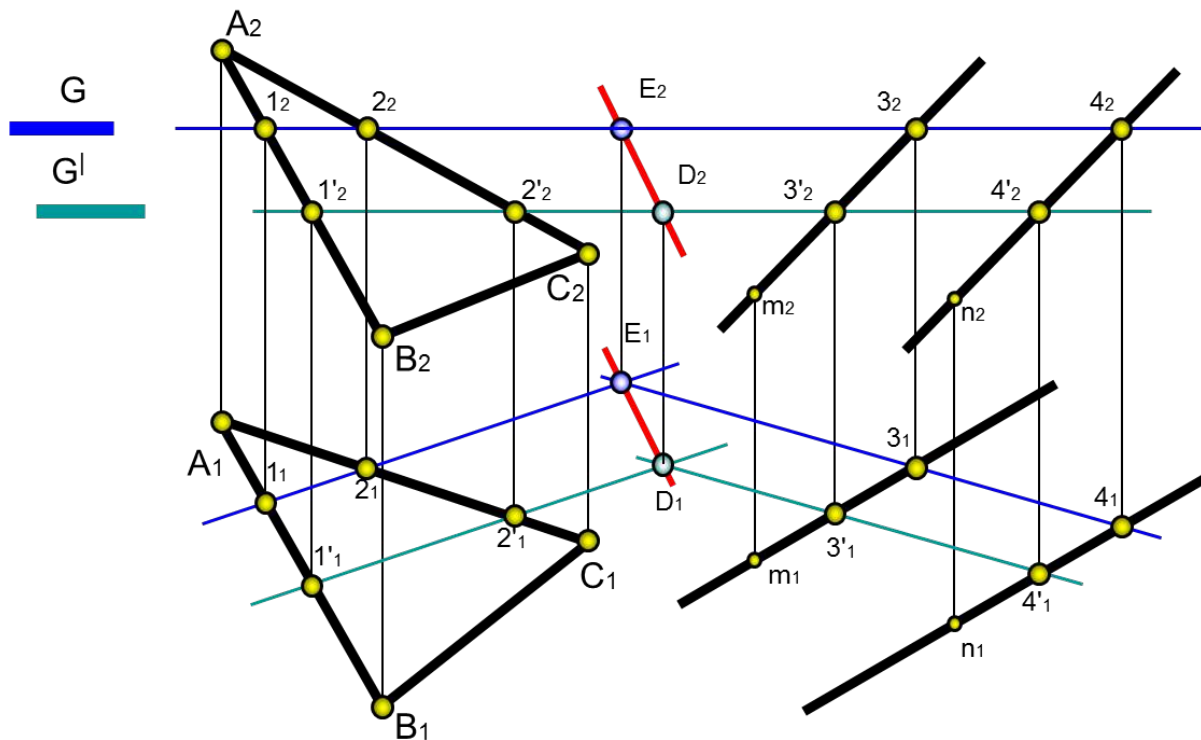


Fig. 8.2

Problems

8.1. Points $A(80,15,20)$, $B(60,35,30)$, $C(35,5,5)$ and $K(10,40,35)$. Through the point K draw a plane, parallel to the plane ABC .

8.2. Points $A(45, _, 35)$, $B(10, _, 30)$, $C(30,15,10)$, and $K(90,20,25)$, $M(50,45,45)$, $N(50,10,5)$. Complete orthographic drawing of the triangle ABC , if plane ABC is parallel to the plane KMN .

8.3. Points $A(125,35,25)$, $B(90,60,5)$, $C(110,5,40)$, $D(75,30,20)$, $M(55,30,10)$, $N(35,10,35)$, $L(10,55,20)$ and $K(35,30,50)$. Through the point K draw a plane, perpendicular to the planes NML and $ABCD$, where $ABCD$ is represented by two parallel lines AB and CD .

8.4. Points $A(10,30,20)$, $B(35,50,55)$, $C(95,35,35)$, $D(75,10,55)$ and $E(50,25,10)$. Through the line AB draw a plane, perpendicular to the plane CDE .

8.5. Points $A(60,30,35)$, $B(10, _, 45)$ and $C(30,5,5)$. Complete projections of the vertex B of isosceles triangle ABC , where AC is the base of the triangle.

8.6. Points $A(65,25,10)$, $B(35,50,30)$, $C(20,10,40)$, $K(55,25,35)$, $L(25,50,15)$ and $M(10,20,5)$. Construct line of intersection between two frontally

perpendicular planes ABC and KLM. Determine visibility of plane segments.

8.7. Points A(65,5,25), B(50,15,50), C(20,35,15), K(55,40,5), L(40,30,45) and M(10,10,25). Construct line of intersection between two horizontally perpendicular planes ABC and KLM. Determine visibility of plane segments.

8.8. Points A(50,20,45), B(5,15,30), C(25,50,5), K(75,20,20), L(55,55,30) and M(15,5,50). Construct line of intersection between frontally perpendicular plane KLM and oblique plane ABC. Determine visibility of plane segments.

8.9. Points A(65,25,35), B(40,5,45), C(25,50,15), K(60,45,20), L(0,15,50) and M(20,25,5). Construct line of intersection between horizontally perpendicular plane KLM and oblique plane ABC. Determine visibility of plane segments.

8.10. Points A(115,20,20), B(90,0,50), C(70,50,5), K(60,20,10), L(30,45,50) and M(5,10,35). Construct line of intersection between planes KLM and ABC. Determine visibility of plane segments.

8.11. Points A(75,30,30), B(50,5,55), C(20,55,10), K(65,50,10), L(30,55,60) and M(5,5,30). Construct line of intersection between planes KLM and ABC. Determine visibility of plane segments.

9. POLYHEDRONS

Polyhedron is a solid figure, bounded by plane polygons. These polygons are called Faces of a polyhedron. Faces of a polyhedron intersect in straight line segments called Edges. Edges meet in points called Vertices (Vertices) of a polyhedron. Multiple faces form polyhedral surface of a polyhedron.

All the vertices and edges of a polyhedron form the mesh of the solid. To construct orthographic view of a polyhedron means to construct orthographic view of the mesh of the polyhedron. The Mesh completely defines a polyhedron and is called Determinant of a polyhedron.

Regular polyhedrons

A Regular polyhedron is a polyhedron whose faces are all regular polygons which are identical in both shape and size. There are two particular spheres associated with any regular polyhedron.

The first is the circum-sphere. This is the sphere which fits around the outside of the polyhedron so as to touch all of its vertices.

The second is the in-sphere. This is the sphere which fits inside the polyhedron so as to touch all of its faces.

There are five regular polyhedrons: tetrahedron (4 triangles), cube (6 squares), octahedron (8 triangles), dodecahedron (12 pentagons), icosahedron (20 triangles) (Fig. 9.1).

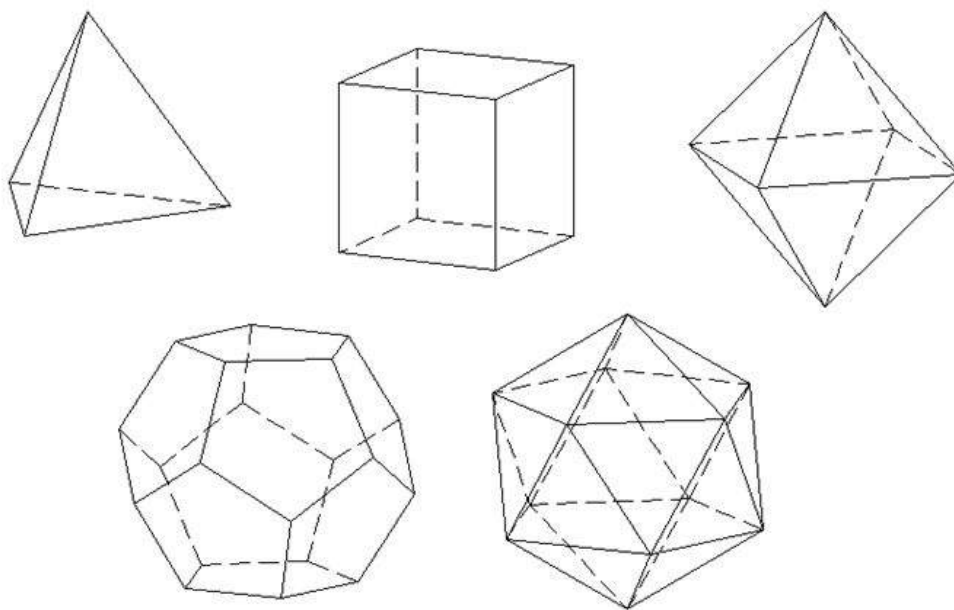


Fig. 9.1

If no face of a polyhedron can cut it on extension, then polyhedron is convex, otherwise it is concave.

For every convex polyhedron ratio between faces, edges and vertexes can be defined by Euler formula

$$F - E + V = 2,$$

where F - number of faces;

E - number of edges;

V - number of vertexes.

The most widespread polyhedrons are pyramids and prisms (Fig. 9.2).

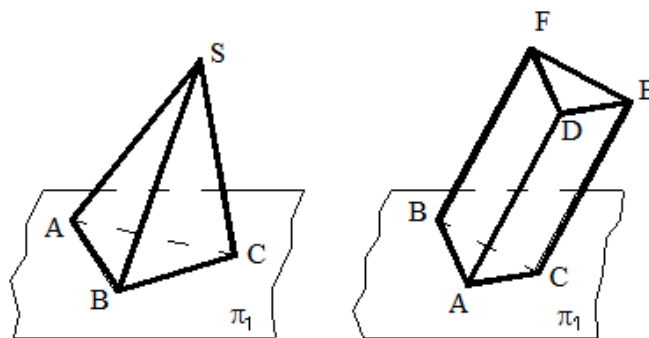


Fig. 9.2

Pyramid is a polyhedron formed by connecting a polygonal base and a point, which doesn't lie in the base plane (Fig. 9.3). This point is called the apex of a pyramid. Each base edge and apex forms a triangle, which is called lateral face.

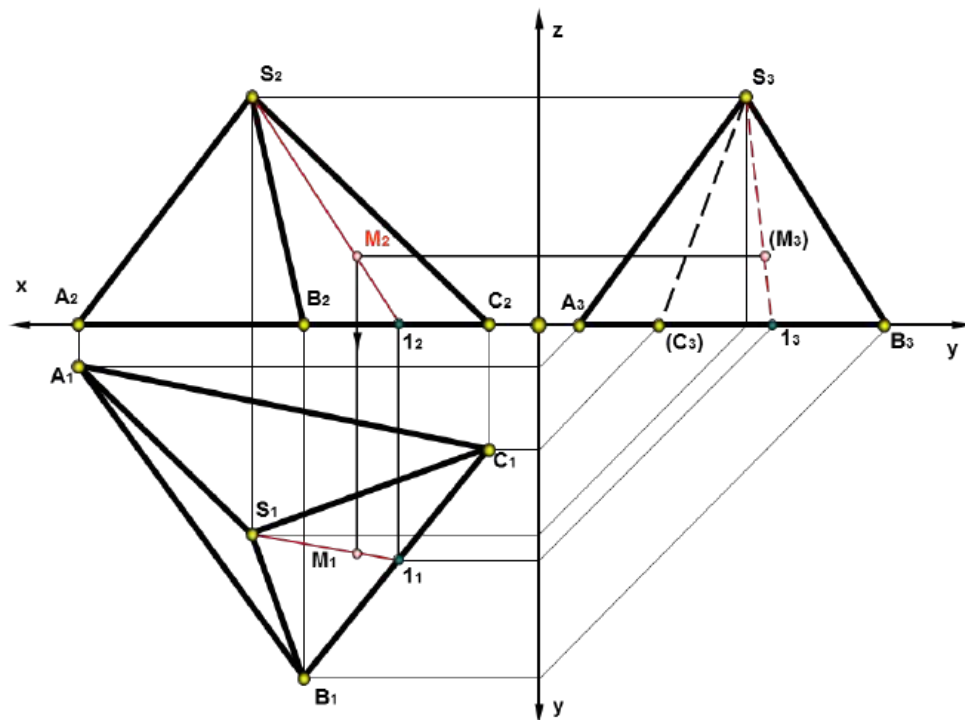


Fig. 9.3

Prism is a polyhedron made of polygonal bases and lateral faces, joining corresponding sides (Fig. 9.4). Thus these lateral faces are parallelograms. All cross-sections parallel to the base faces are of the same shape and size.

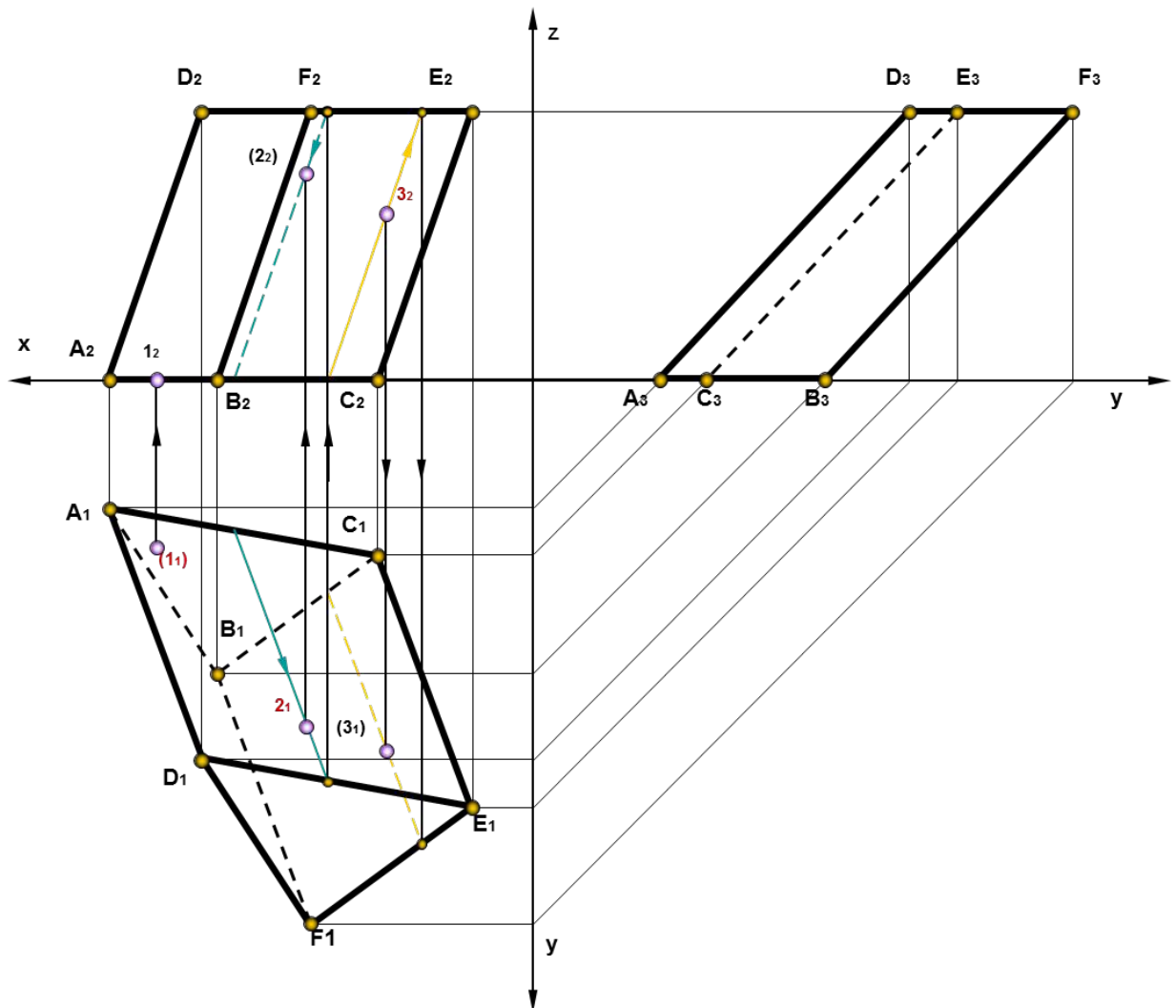


Fig. 9.4

Projections of points, which belong to the faces of polyhedrons, can be defined using proper projections of construction lines, which pass through that points and lie in the proper faces of polyhedrons (see Figs. 9.3, 9.4).

Visibility in polyhedrons

Crossing points are commonly used to define visibility of edges and faces in orthographic views of a polyhedron. There are also four main rules, which can help to definite visibility in polyhedrons:

1. Outline of the figure in a view is always visible.
2. If two edges intersect inside outline, then one is visible, and the other is invisible (hidden).

3. If three edges meet in one point (vertex) inside the contour, then they are all visible or they are all hidden.
4. If sequence of vertexes for the polyhedron's face is the same on orthographic views, then visibility is the same, otherwise it is different.

Problems

- 9.1. Pyramid SABCD: apex $S(30,30,45)$ and base $A(70,35,0)$, $B(55,15,0)$, $C(5,5,0)$, $D(25,55,0)$. Construct three orthographic views of the pyramid. Determine visibility of edges of pyramid. Complete projections of the points $K(20, _, 15)$ and $M(35, 20, _)$, which belong to pyramid's surface.
- 9.2. Pyramid SABCDE: apex $S(5,55,60)$ and base $A(70,35,0)$, $B(50,5,0)$, $C(10,15,0)$, $D(15,35,0)$, $E(55,60,0)$. Construct three orthographic views of the pyramid. Determine visibility of pyramid's edges. Construct orthographic views of the pyramid $S'AB CDE$, if ratio between volumes of SABCDE and $S'AB CDE$ is equal to 5:7.
- 9.3. Points $A(85,40,0)$, $B(65,15,0)$, $C(50,35,0)$ and $A'(40,60,50)$. Construct three orthographic views of triangular oblique prism $ABCA'B'C'$, if ABC is the base and AA' is lateral edge of the prism. Determine visibility of the edges. Complete projections of the point $K(55,45, _)$, which belongs to the surface of the prism.
- 9.4. Points $A(85,40,0)$, $B(65,10,0)$, $C(50,35,0)$ are vertexes of the base of oblique triangular prism $ABCA'B'C'$. Lateral edges of the prism are frontal line segments. Angle between horizontal principal plane and lateral edges is 60° . Height of the prism is 50mm. Construct three orthographic views of triangular oblique prism $ABCA'B'C'$ and determine visibility of edges.

10. TRANSFORMATION OF A DRAWING

Some problems can be solved easier if object has a particular location relative to the plane of projection. For instance, if a straight line segment is parallel to the plane of projection, it projects to a true size. Otherwise, to find its true length we need to use method of a right-angular triangle. To achieve this we can apply methods called Transformation of a Drawing.

There are three basic ways to transform a drawing:

1. Projection to auxiliary planes (Replacement of planes of projection).
2. Revolution of a model about axes perpendicular to a plane of projection.
3. Planar parallel motion (revolution of a model about axes without indicating axes in the drawing).

Basic problems which can be solved by Transformation of a drawing:

- oblique line segment transforms to a line segment parallel to the plane of projection;
- oblique line segment transforms to a line segment perpendicular to the plane of projection;
- oblique plane segment transforms to a plane segment perpendicular to the plane of projection;
- oblique plane segment transforms to a plane segment parallel to the plane of projection.

Projection onto auxiliary planes

In this method location of an object doesn't change while one of planes of projection is replaced by a new one, perpendicular to the other plane of projection.

Properties of transformation:

1. New projectors are perpendicular to new axes;
2. Distances between new projections and new axes are equal to distances between replaced (old) projections and replaced axes.

Projection of a line segment on auxiliary plane

Points A and B are given by their projections in Π_2/Π_1 planes (Figs. 10.1, 10.2). Construct projections of a point A, if the plane Π_2 is replaced by a new vertical plane Π'_2 .

Horizontal projection A_1 doesn't change because location of Π_1 is not changed. To determine new frontal projection A'_2 from A_1 point draw perpendicular projector to Π'_2 plane (to x_1 -axis) and on its extension protract distance, equal to the height of the point A (z- coordinate $A_x A_2 = A_{x1} A'_2$). A'_2 – new frontal projection of the point A. Similar constructions should be made with the point B.

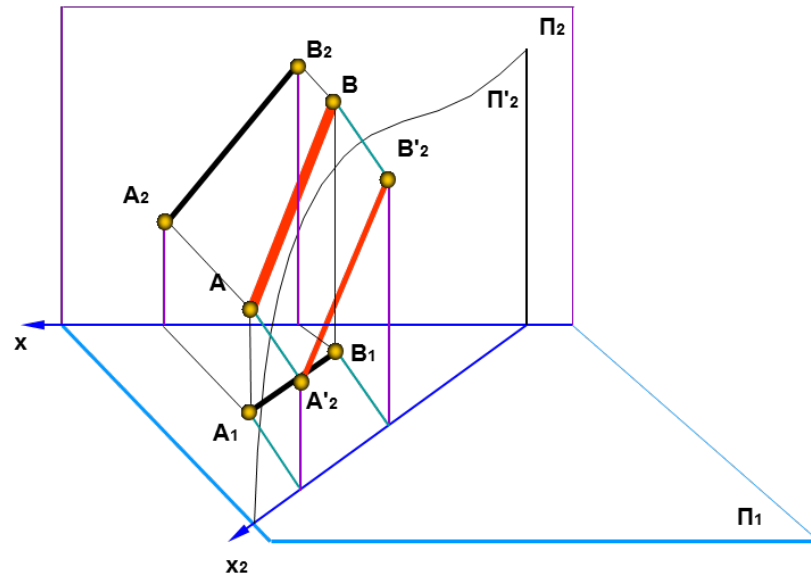


Fig. 10.1

$A'_2 A_{x1} = A_2 A_x$. $A_2 A_x$ is Z-coordinate of a point A (Z_A). It means that if frontal plane of projection is replaced, Z-coordinates of entire figure don't change.

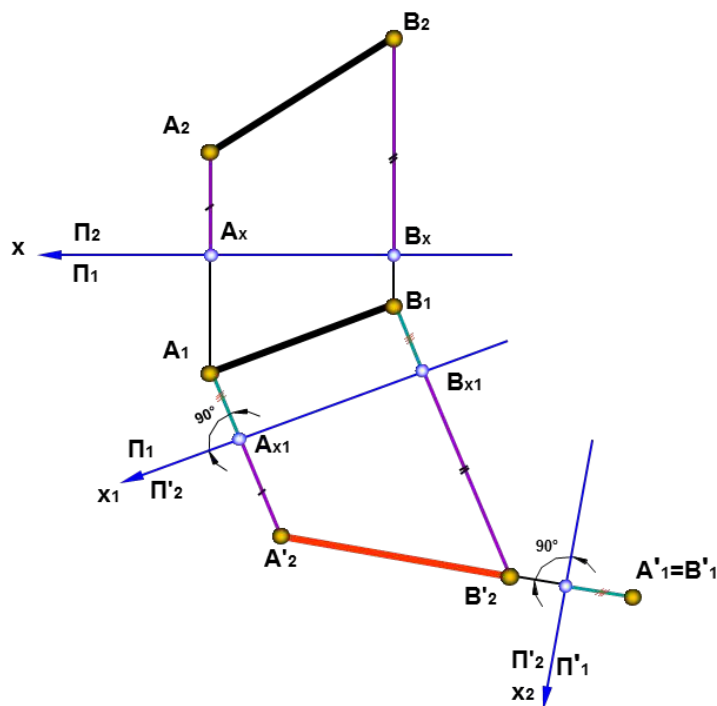


Fig. 10.2

By this method oblique line segment transforms to a line segment parallel to the plane of projection and in succession perpendicular to the other plane of projection.

Algorithm to transform a line segment:

3. Implement new frontal plane (primary auxiliary plane) $\Pi'_2 \parallel [AB]$ to transform A_1B_1 to a line parallel to a principal plane.
4. Construct frontal projection of $[A'_2 B'_2]$. Z-coordinates of the points A and B don't change.
5. Implement new horizontal plane (secondary auxiliary plane) $\Pi'_1 \perp [A'_2 B'_2]$.
6. Construct horizontal projection of a line $[A'_1 B'_1]$ in a new system Π'_1 .

Projection of a plane segment on auxiliary plane

By this method (Figs. 10.3, 10.4) oblique plane segment transforms into a plane segment perpendicular to the plane of projection and in succession parallel to the other plane of projection.

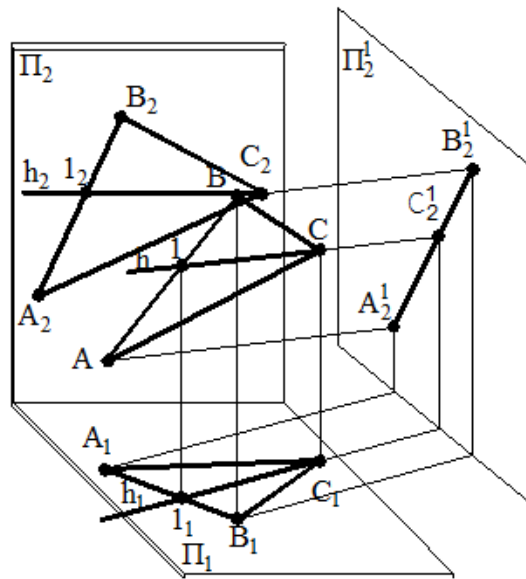


Fig. 10.3

Algorithm to transform oblique plane segment to a plane segment parallel to a principal plane of projection:

1. Draw horizontal line in the plane (ABC) – h_2 .
2. Construct horizontal projection of a horizontal line – h_1 .
3. To implement a primary auxiliary plane Π'_2 draw a new axes x'_2 perpendicular to h_1 .

4. Construct new frontal projection of ABC-plane – $A'_2B'_2C'_2$ in a plane Π'_2 .

5. To implement a secondary auxiliary plane Π'_1 draw a new axes x''_1 parallel to $A'_2B'_2C'_2$.

6. Construct new horizontal projection of ABC-plane – $A'_1B'_1C'_1$ in a plane Π'_1 .

$A'_1B'_1C'_1$ is a true size (TS) of ABC-plane.

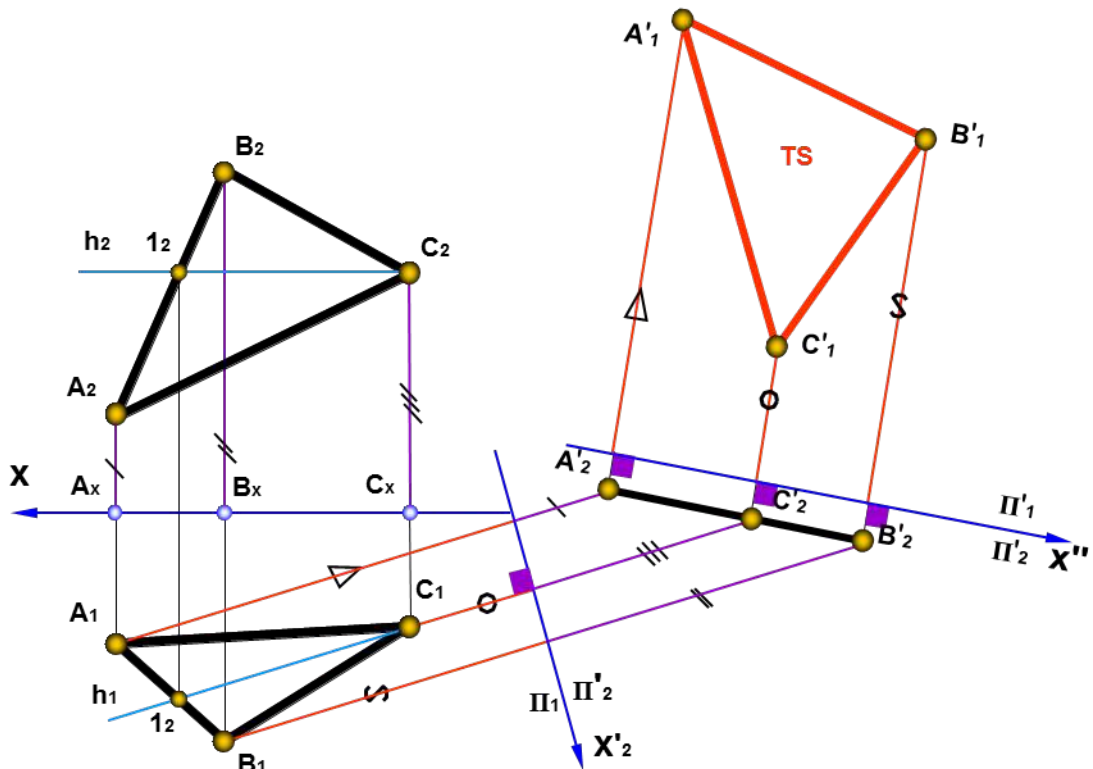


Fig. 10.4

Problems

10.1. Points $A(40,10,40)$ and $B(10,30,10)$. Project line segment AB on the auxiliary plane so that the line AB would be horizontal (parallel to P_1).

10.2. Points $A(40,40,10)$ and $B(10,10,30)$. Project line segment AB on the auxiliary plane so that the line AB would be frontal (parallel to P_2).

10.3. Points $A(40,20,10)$ and $B(10,5,30)$. Project line segment AB on the auxiliary planes so that the line AB would be horizontally projecting (perpendicular to P_1).

10.4. Points $A(70,15,10)$, $B(50,30,40)$ and $C(10,5,10)$. Project plane segment ABC on the auxiliary plane so that the plane ABC would be horizontally projecting (perpendicular to P_1).

- 10.5. Points $A(60,10,25)$, $B(30,35,40)$ and $C(10,10,10)$. Project plane segment ABC on the auxiliary planes so that the plane ABC would be horizontal (parallel to P_1).
- 10.6. Points $A(35,25,30)$ and $B(10,5, _)$. Construct orthographic projections of the AB line segment, if its length is equal to 35mm.
- 10.7. Points $A(35, _, 25)$ and $B(10,20,5)$. Construct orthographic projections of the AB line segment, if its length is equal to 35mm.
- 10.8. Points $A(50,30,10)$, $B(20,10,45)$ and $C(5,15,15)$. Construct two orthographic views of the line segment CK, perpendicular to the line AB.
- 10.9. Points $A(40,40,5)$, $B(10,10,20)$ and $C(20, _, 5)$. Complete projections of the points C, spaced from the line AB on 20 mm.
- 10.10. Points $A(70,30,30)$, $B(45,5,45)$, $C(55,15,15)$, $A'(30,30,20)$, $B'(5,5,35)$, $C'(15,15,5)$ are vertexes of oblique triangular prism, where ABC and $A'B'C'$ are bases. Project triangular prism ABCA'B'C' on the auxiliary plane so that the base ABC would be frontal (parallel to the plane P_2).
- 10.11. Points $A(50,20,30)$, $B(10,10,20)$, $C(40,25,10)$ and $D(20,25,45)$. Find points, which belong to the line AB and allocated at the distance of 20mm from the line CD.

11. CURVE LINES

Lines and Curves are constituent elements for both engineering sketching and computer modeling.

A curve can be represented as:

1. A trajectory of a moving point;
2. An intersection between a plane and a surface;
3. An intersection between two surfaces;

Plane Curve – all the points of the curve lie in one plane.

Space Curve – a curve line, which doesn't lie entirely in one plane.

To represent a plane curve on orthographic drawing it is sufficient to have its one projection and three points of the other projection (Fig. 11.1).

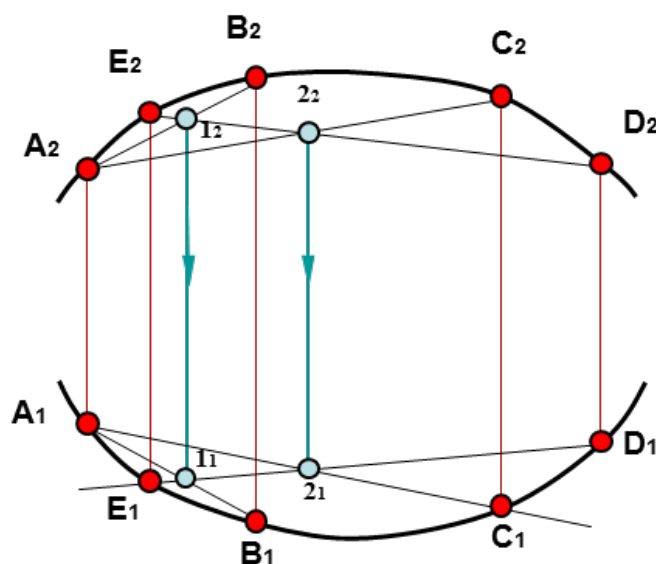


Fig. 11.1

Cylindrical helix

Cylindrical helix can be considered as a result of point's moving on the surface of revolution.

Cylindrical (Ordinary) helix is a space screw curve, formed by the point, which moves along straight line, and rotates around some axis (Fig. 11.2).

Distance along the axis after one complete revolution of the point is called lead of a helix.

Helixes can be right hand or left hand depending on direction of rotation of a point.

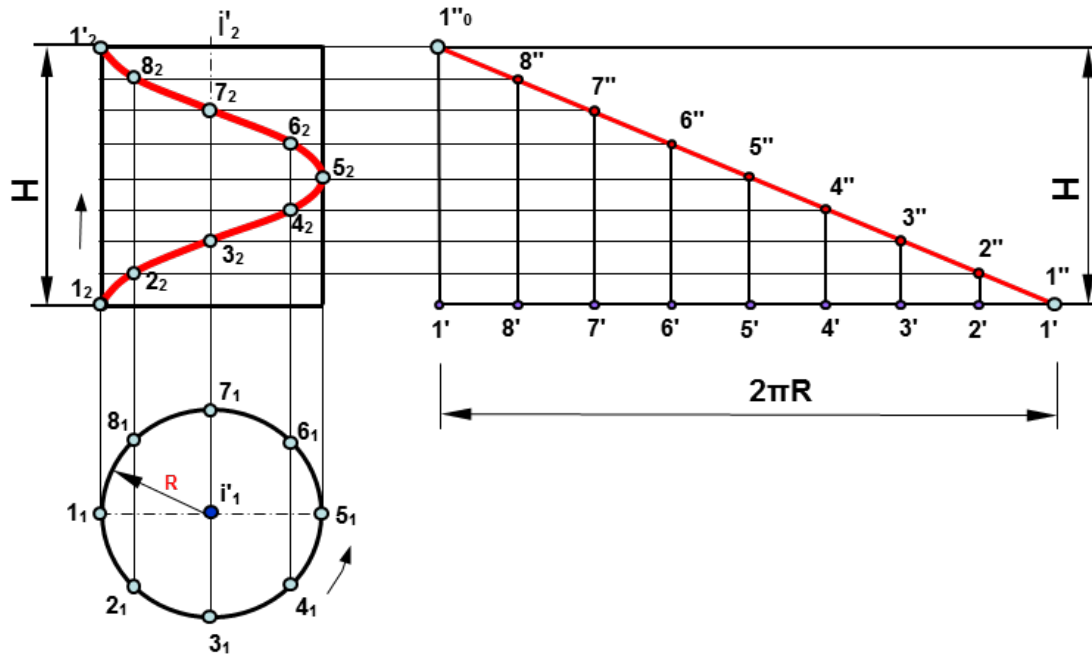


Fig. 11.2

Algorithm to construct orthographic drawing of an ordinary helix:

1. Draw orthographic views of a right cylinder of revolution.
2. Bisect base of the cylinder (circle) on several number of segments.
3. Bisect lead (pitch) of a helix on the same number of segments (on the frontal view).
4. Construct proper projectors from the circle and horizontal line segments from bisecting points.
5. Label points $1_2, 2_2, 3_2, 4_2, \dots 8_2$, where corresponding vertical projectors and horizontal line segments are intersected.
6. Join obtained points by smooth French curve and obtain frontal view of an ordinary helix.

Problems

11.1. Points $A(125,55,10)$, $B(120,50, _)$, $C(110,50, _)$, $D(100,55, _)$, $E(90,60, _)$, $F(80,65, _)$, $G(65,70, _)$, $H(50,70,60)$, $I(35,65, _)$,

J(25,55,___), K(20,40,___), L(25,25,___), M(35,15,30). Construct orthographic projections of the plane curve line, which passes through these points.

11.2. Center of the circle, is allocated in the point O(40,40,40). The circle belongs to the profile projecting plane (perpendicular to P_3). Angle between the plane of the circle and horizontal principal plane is 60° . Radius of the circle is 30mm. Construct frontal and horizontal projections of the circle.

11.3. Points A(70,___,15) and B(10,___,45). AB is the major axis of the ellipse. CD is the minor axes of the same ellipse, and it's two times smaller than the major axis. Points A, B, C and D belong to the plane, which passes through the OX-axis. Angle between ABCD-plane and horizontal principal plane is 60° . Construct horizontal and frontal projections of the ellipse.

11.4. Points $A_1(80,10)$, $B_1(10,10)$, $C_1(30,50)$ and $D_1(100,50)$. $A_1B_1C_1D_1$ is horizontal projection of the square ABCD. Construct horizontal projection of the circle, inscribed in the square ABCD.

11.5. Points S(55,15,10), O(25,___,55), A(50,40,65), B(15,15,___). S – apex, SO – axis of the parabola. Construct horizontal and frontal projections of the parabola, if it passes through the points A and B.

11.6. Axis of the cylinder is perpendicular to the horizontal principal plane. Radius of the circle, which is the base of the cylinder, is 35mm. Construct orthographic projections of cylindrical helix, based on the given cylinder. Lead of the helix is 80mm.

11.7. Points S(40,40,80), O(40,40,0). S – apex of the cone. O – center of the circle, which is the base of the cone. Radius of the circle is 35mm. Construct orthographic projections of conical helix, based on the given cone. Lead of the helix is 80mm.

12. RULED SURFACES

Surface of geometric solid is multiple of boundary points of a given body.

To specify a surface in a drawing means to indicate the conditions enabling us to construct each point of this surface. Surfaces can be represented analytically or graphically. Graphic representation includes framing or kinematic way.

Kinematic surface represents a locus of lines moving in space according to a certain law. These lines, which may be straight or curved, are called *generating lines (generatrices)*.

In general, there is a variety of ways for generating a certain surface. It is desirable to select those ways and shapes of the generating lines which are most simple and convenient both for representing the required surface and for solving the problems associated with it.

Determinant of kinematic surface is minimum number of parameters and constraints which univocally represent given surface.

A surface which can be generated by a straight line is called **ruled surface**. Hence, ruled surface represents a locus of straight lines.

Conical surface

A surface generated by a straight line passing through a fixed point and through all the points (in succession) of a curved directing line is called Conical surface (Fig. 12.1). This line is also called **directrix** or **guide line**. The fixed point S is called the **vertex** of a conical surface.

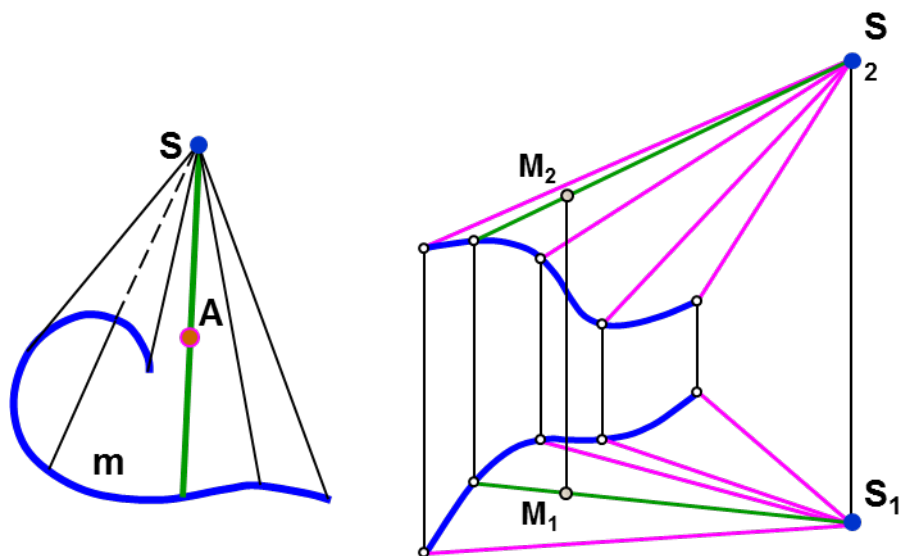


Fig. 12.1

Cylindrical surface

If a vertex of a conical surface is moved to infinity, then conical surface turns into cylindrical.

A **cylindrical surface** is generated by a straight line which is parallel in all its positions to a given straight line and passes in succession through all points of a curved **directing line**.

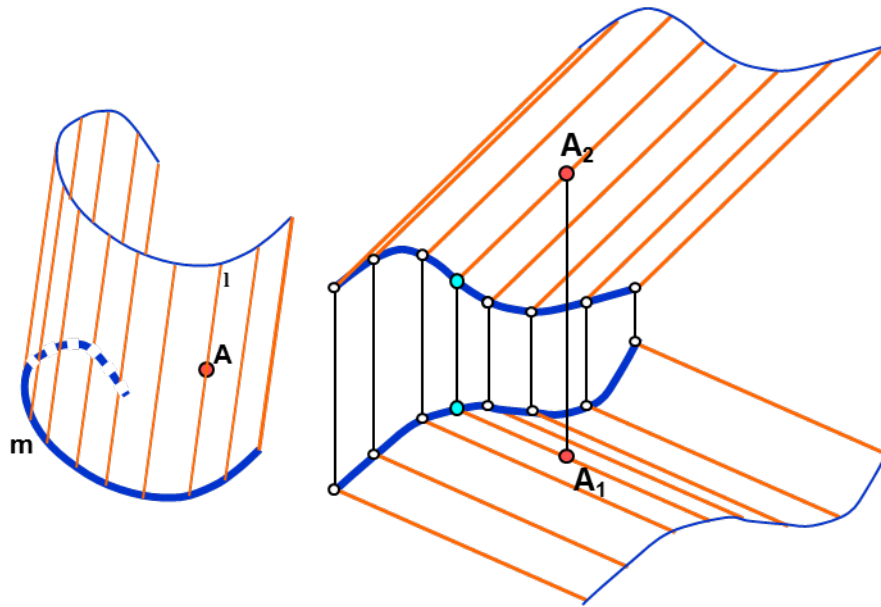


Fig. 12.2

Problems

12.1. Points $S(0,0,50)$, $A(35,20,0)$, $K(45,30,10)$, $L(60,35,15)$, $M(75,35,10)$, $N(85,30,5)$, $B(100,10,15)$, and $C(50, _, 25)$, $D(15, _, _)$. Construct orthographic projections (outline and a few intermediate locations of generatrix) of the ruled conical surface. S – apex of the conical surface, SA – generatrix (generating line), $AKLMNB$ – directrix (smooth curve guide line). Curve CD belongs to the conical surface. Frontal projection of the curve CD is a straight line C_2D_2 . Point D belongs to the line SA . Construct horizontal projection of the curve line CD .

12.2. Points $S(0,20,60)$, $A(40,35,0)$, $O(70,35,0)$, $C(80, _, 5)$, $D(25, _, _)$ and $E(35, _, 30)$. Construct orthographic projections (outline and a few intermediate locations of generatrix) of the ruled conical surface. S – apex of the conical surface, SA – generatrix (generating line); directrix (guide line) of the surface is horizontally located circle of radius 30mm. O – center of the circle. Curve CDE belongs to the conical surface. Frontal projection of the curve CDE consists of two straight line segments: C_2D_2 and D_2E_2 .

Point D belongs to the line SA. C_2 is invisible. Construct horizontal projection of the curve line CDE. Determine visibility of segments.

12.3. Points A(0,15,35), B(40,5,0), K(55,10,0), L(70,20,0), M(70,35,0), C(60,45,0) and D(20,20,__). Construct orthographic projections (outline and a few intermediate locations of generatrix) of the ruled cylindrical surface. AB – generatrix (generating line) of the cylindrical surface, SA – generatrix; BKLMC – directrix (smooth curve guide line). Curve CD belongs to the given cylindrical surface. Horizontal projection of the curve CD is a straight line C_1D_1 . Construct frontal projection of the curve line CD and determine visibility of the curve.

12.4. Points A(70,35,50), B(10,35,0), O(25,__,0), C(85,__,40) and D(30,__,__). Construct orthographic projections (outline and a few intermediate locations of generatrix) of the ruled cylindrical surface. AB – generatrix (generating line) of the cylindrical surface; directrix (guide line) of the surface is horizontally located circle of radius 20mm. O – center of the circle. Curve CD belongs to the cylindrical surface. Frontal projection of the curve CD is a straight line segment C_2D_2 . Point D belongs to the line AB. C_2 is invisible. Construct horizontal projection of the curve line CDE. Determine visibility of segments.

13. SURFACES OF REVOLUTION

A *surface of revolution* is generated by the revolution of a curved-line or straight-line generatrix about a fixed straight line called the *axis of the revolution*.

Determinant of the surface of revolution: generatrix and axis of revolution. Each point of the generatrix describes a circle (Fig. 13.1). Hence, a plane perpendicular to the axis of revolution cuts this surface in circles. Such circles are called *parallels*. The biggest parallel is called the *equator*, the smallest parallel is called the *neck* of a surface. Neck and equator project to outlines.

A plane passing through the axis of a surface of revolution is termed a *meridian plane*. The line where a meridian plane intersects a surface of revolution is called the *meridian of the surface*.

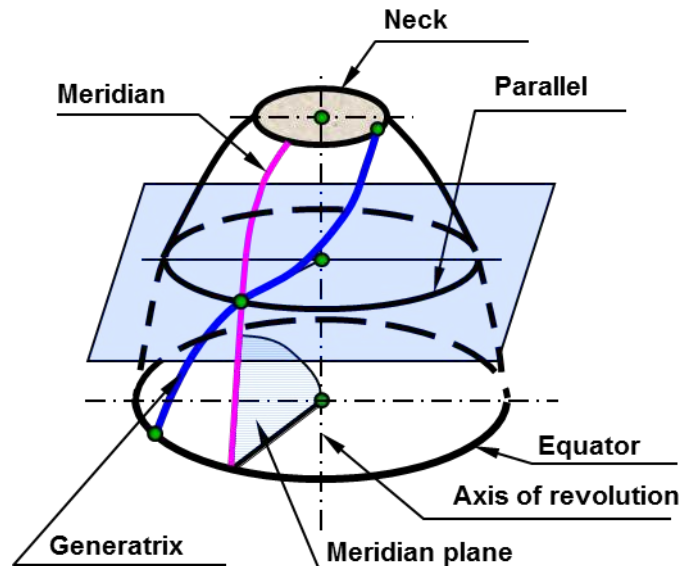


Fig. 13.1

There are several types of the surfaces of revolution, which can be obtained if a generatrix is a straight line, depending on relative position of the generatrix and axis of revolution. If generatrix and axis of revolution are intersecting lines, then obtained figure is called ***cone of revolution***. If generatrix and axis of revolution are parallel lines, then obtained figure is called ***cylinder of revolution***. If generatrix and axis of revolution are skew (crossing) lines, then obtained figure is called ***single sheet hyperboloid of revolution***.

If generatrix is a circle, then it's possible to obtain the following surfaces of revolution: a sphere, open or close torus and globoid.

Single sheet hyperboloid of revolution has hyperbola as a meridian and can be constructed if hyperbola revolves about its imaginary axis, or if straight line - generatrix and axis of revolution are skew (crossing) lines (Fig. 13.2).

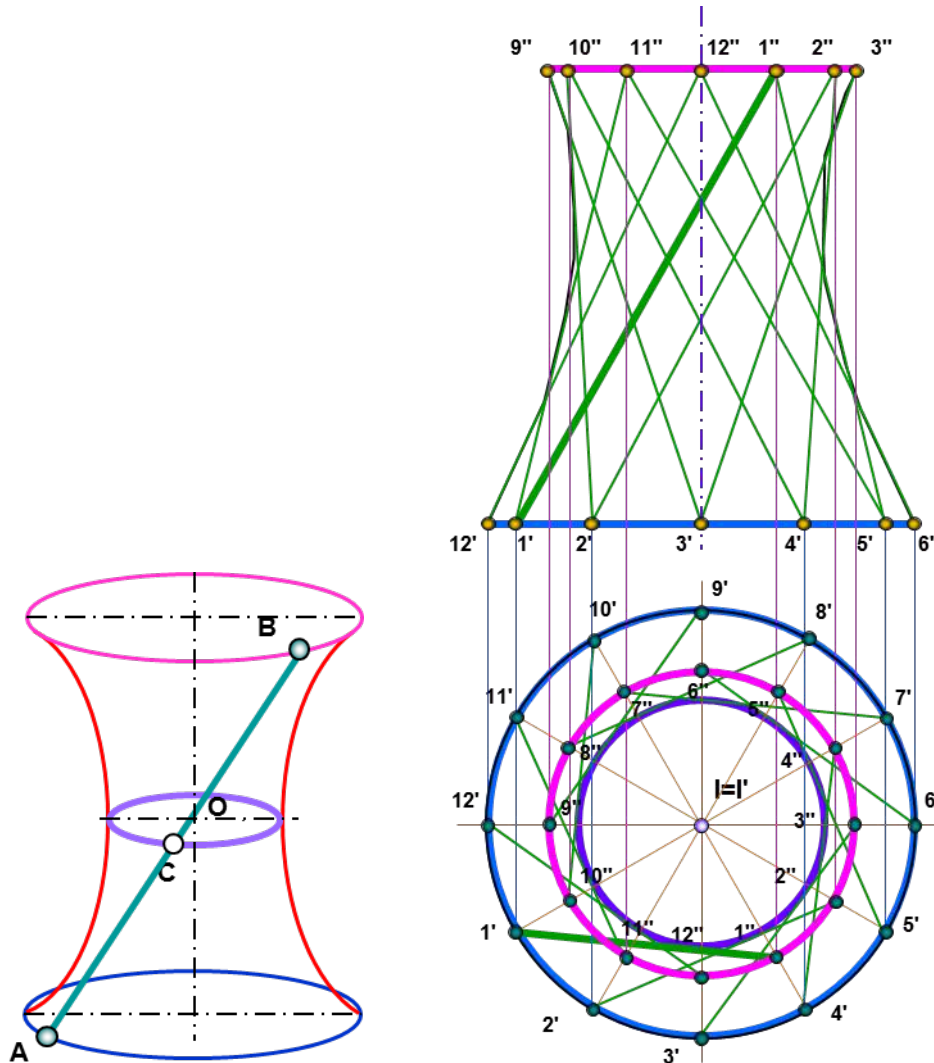


Fig. 13.2

Problems

13.1. Points $S(30,30,60)$, $A(55,30,10)$, $I(30,30,65)$, $J(30,30,5)$, $K(40,50, _)$ and $L(15,15, _)$. Construct orthographic projections of the cone of revolution, if straight line SA – generatrix of the cone, IJ – axis of revolution. Complete projections of the curve KL , which belongs to the cone surface. Projection K_1L_1 is a straight line.

13.2. Points $A(10,35,55)$, $B(10,35,10)$, $I(25,25,60)$, $J(25,25,5)$, $K(10,35,45)$ and $L(_,10,15)$. Construct orthographic projections of the cylinder of revolution, if straight line AB – generatrix of the cylinder, IJ – axis of

revolution. Complete projections of the curve KL, which belong to the cylinder surface. Projection K_3L_3 is a straight line.

13.3. Points $S(25,25,50)$, $A(35,40,5)$, $I(25,25,55)$, $J(25,25,0)$, $K(_,_,30)$ and $L(_,15,10)$. Construct orthographic projections of the cone of revolution, if straight line SA – generatrix of the cone, IJ – axis of revolution. Complete projections of the curve KL, which belong to the cone surface. Point K belongs to SA. Projection K_3L_3 is a straight line. Projection L_3 is visible.

13.4. Points $A(55,20,5)$, $B(10,20,5)$, $I(60,20,20)$, $J(5,20,20)$, $K(45,_,10)$ and $L(20,_,35)$. Construct orthographic projections of the cylinder of revolution, if straight line AB – generatrix of the cylinder, IJ – axis of revolution. Complete projections of the curve KL, which belongs to the cylinder surface. Projection K_2L_2 is a straight line.

13.5. Points $O(30,30,30)$, $K(40,50,_)$ and $L(15,15,_)$. Construct orthographic projections of the sphere of a radius 25mm, if point O – center of the sphere. Complete projections of the curve KL, which belongs to the sphere surface. Projection K_1L_1 is a straight line.

13.6. Points $A(45,40,50)$, $B(65,40,30)$, $C(45,40,10)$, $I(35,40,55)$, $J(35,40,5)$, $K(55,30,_)$ and $L(30,65,_)$. Construct orthographic projections of the torus, if ABC – circular arc – generatrix of the torus, IJ – axis of revolution. Complete projections of the curve KL, which belongs to the torus surface. Projection K_1L_1 is a straight line.

13.7. Points $A(40,15,_)$, $B(15,45,_)$, $C(33,_,20)$, $D(27,_,30)$, $I(30,30,40)$, $J(30,30,10)$ and $O(45,30,25)$. Construct orthographic projections of the torus, if generatrix of the torus is a circle, placed in a frontal plane (parallel to frontal principal plane). Radius of the circle is 20mm. Point O – center of the circle. IJ – axis of revolution. Complete projections of the points A, B, C and D, which belong to the torus surface. Projections B_1 , C_2 and D_2 are invisible.

13.8. Points $A(75,50,80)$, $B(60,50,65)$, $C(75,50,50)$, $D(95,50,30)$, $E(75,50,10)$, $I(50,50,85)$, $J(50,50,5)$, $K(40,_,75)$ and $L(80,_,25)$. Construct orthographic projections of the surface of revolution. ABCDE – generatrix. ABC – circular arc, CDE – circular arc. IJ – axis of revolution. Complete projections of the curve KL, which belongs to the surface of revolution. Projection K_2L_2 is a straight line.

13.9. Points $A(40,50,60)$, $B(10,20,10)$, $I(35,35,65)$, $J(35,35,5)$, $K(50,_,15)$ and $L(30,_,55)$. Construct orthographic projections of the surface of

revolution. AB – generatrix – straight line. IJ – axis of revolution. Complete projections of the curve KL, which belongs to the surface of revolution. Projection K_2L_2 is a straight line. Determine visibility of the curve.

13.10. Points A(50,20,10), B(30,50,60), I(25,35,65), J(25,35,5), K(40,__,15) and L(15,__,55). Construct orthographic projections of the surface of revolution. AB – generatrix – straight line. IJ – axis of revolution. Complete projections of the curve KL, which belongs to the surface of revolution. Projection K_2L_2 is a straight line. Determine visibility of the curve.

14. A PLANE AND A SURFACE. SECTIONS

To construct section of a surface by a plane means to define set of points which simultaneously belong to a plane and a surface. All these points belong to a plane and form plane figure called section.

Combined figure sectioned by a plane. The model (Fig. 14.1) consists of horizontally projecting (vertical) cylinder of revolution, vertical square prism and frontally projecting triangular prism. It's necessary to construct sections of this figure by cutting planes A-A and B-B.

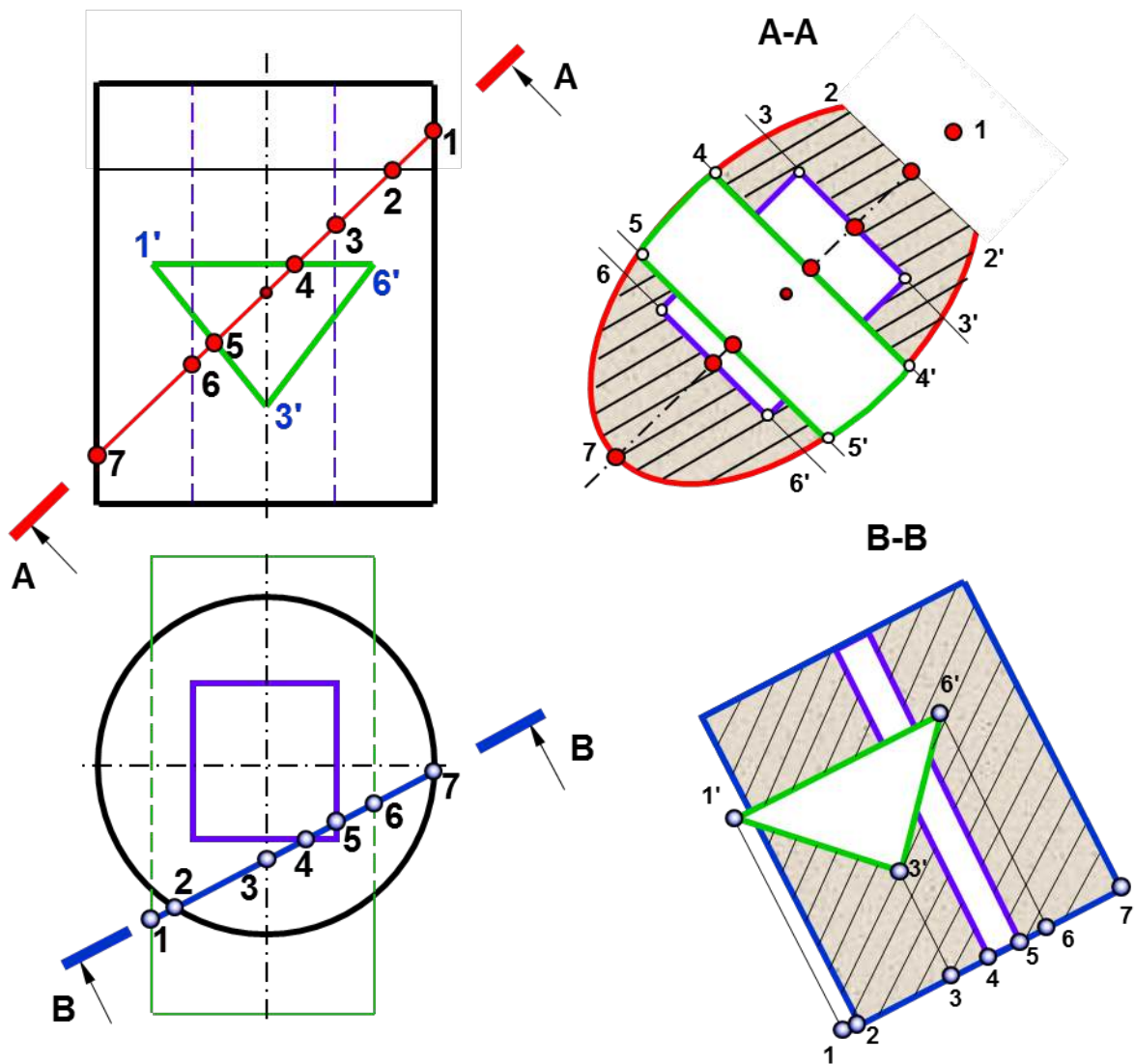


Fig. 14.1

Algorithm to construct a section:

1. Designate all character points of intersection between cutting plane and surfaces of the figure.
2. Locate axes of section 1 – 7 parallel to a cutting line. Mark distances between the points in question.

3. Construct true sizes of full sections from corresponding marked points (for example, A-A section – ellipse from cylinder cutting between points 1 – 7, rectangle from pyramid cutting between points 3 – 6, etc.).
4. Designate obtained section according to a standard (ISO, GOST, DSTU, etc.).

Problems

14.1. Points A(65,20,0), B(45,50,0), C(20,5,0), S(10,45,40), M(65,45,__) and N(5,5,__). Horizontally-projecting (horizontally-perpendicular) plane MN intersects pyramid SABC. Construct orthographic projections of the pyramid SABC, sectioned by horizontally-perpendicular plane MN, and true size of the section.

14.2. Points A(65,20,50), B(45,50,50), C(20,5,50), S(10,45,0), M(65,__,15) and N(25,__,60). Frontally-projecting (frontally-perpendicular) plane MN intersects pyramid SABC. Construct orthographic projections of the pyramid SABC, sectioned by frontally-perpendicular plane MN, and true size of the section.

14.3. Points A(65,20,0), B(45,50,0), C(20,5,0), S(10,45,40), M(__,20,35) and N(__,40,0). Profily-projecting (profily-perpendicular) plane MN intersects pyramid SABC. Construct orthographic projections of the pyramid SABC, sectioned by profily-perpendicular plane MN, and true size of the section.

14.4. Points S(30,30,60), O(30,30,0), M(60,__,5) and N(10,__,50). Frontally-projecting (frontally-perpendicular) plane MN intersects cone of revolution. S – apex of the cone, O – center of the cone base, radius of the circle (base cone) is 25mm. Construct orthographic projections of the cone of revolution, sectioned by frontally-perpendicular plane MN, and true size of the section.

14.5. Points S(30,30,60), O(30,30,0), M(50,60,__) and N(5,10,__). Horizontally-projecting (horizontally-perpendicular) plane MN intersects cone of revolution. S – apex of the cone, O – center of the cone base, radius of the circle (base cone) is 25mm. Construct orthographic projections of the cone of revolution, sectioned by horizontally-perpendicular plane MN, and true size of the section.

14.6. Points O(30,30,30), M(60,__,10) and N(10,__,55). Frontally-projecting (frontally-perpendicular) plane MN intersects sphere. O – center of the sphere. Radius of the sphere is 25mm. Construct orthographic projections of sphere, sectioned by frontally-perpendicular plane MN, and true size of the section.

15. A LINE AND A SURFACE. INTERSECTIONS

If a line intersects a surface, then a line and a surface have common point(s). These points are called piercing points (Fig. 15.1, 15.2).

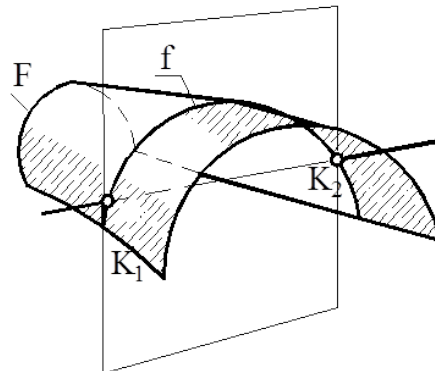


Fig. 15.1

Intersection between AB-line and cylindrical surface. K – piercing point (Fig. 15.2).

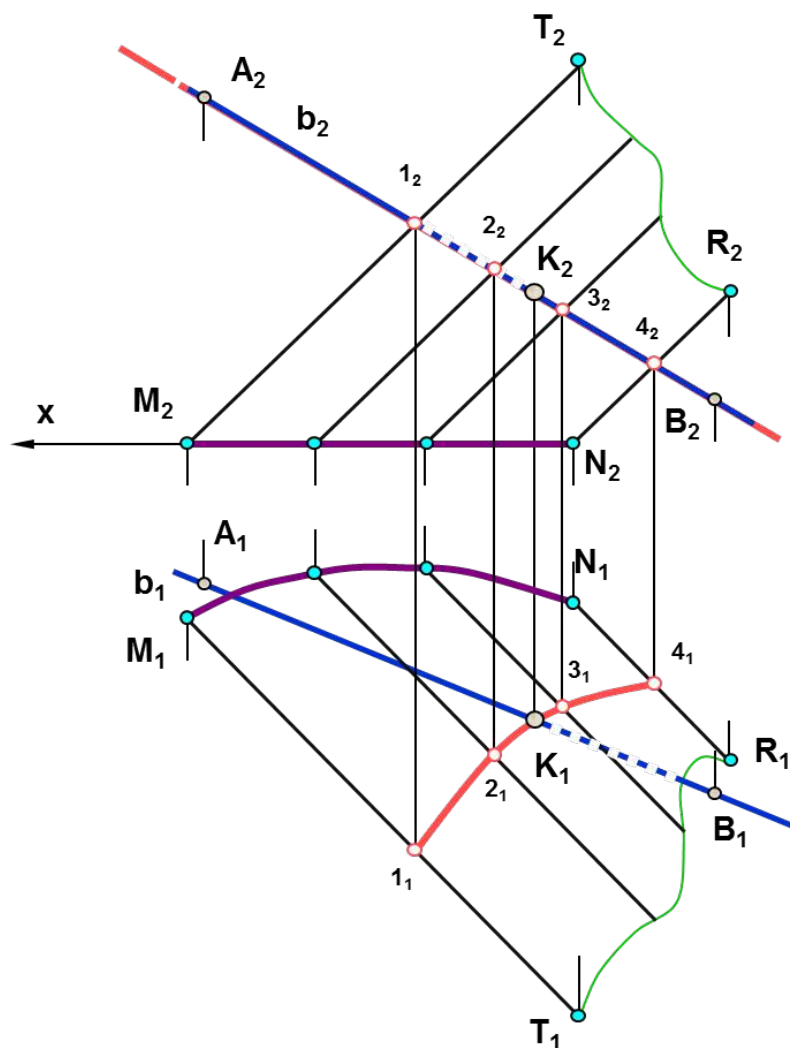


Fig. 15.2

Algorithm to find piercing point:

1. Construct cutting plane which would contain the given line.
2. Find curve of intersection between the surface and cutting plane.
3. Determine point of intersection between obtained curve of intersection and given line.
4. Determine visibility of the line segments.

Problems

15.1. Points $S(30,25,45)$, $A(65,45,0)$, $B(40,5,0)$, $C(5,35,0)$, $K(70,15,5)$ and $M(0,55,30)$. Construct orthographic projections of the pyramid $SABC$ and line KM . S – apex, ABC – base of the pyramid. Determine and construct points of intersection (piercing points) between pyramid $SABC$ and line KM . Indicate visibility of the line segments.

15.2. Points $S(5,25,50)$, $A(65,15,0)$, $B(15,10,0)$, $C(25,45,0)$, $D(50,40,0)$, $K(30,5,10)$ and $M(30,55,10)$. Construct orthographic projections of the pyramid $SABCD$ and line KM . S – apex, $ABCD$ – base of the pyramid. Determine and construct points of intersection (piercing points) between pyramid $SABC$ and line the KM . Indicate visibility of the line segments.

15.3. Points $A(95,35,0)$, $B(75,50,0)$, $C(60,10,0)$, $A'(40,35,55)$, $B'(20,50,55)$, $C'(5,10,55)$, $K(45,25,65)$ and $M(45,25,5)$. Construct orthographic projections of the prism $ABCA'B'C'$ and line KM . ABC – base of the prism. Determine and construct points of intersection (piercing points) between prism $ABCA'B'C'$ and the line KM . Indicate visibility of the line segments.

15.4. Points $A(80,20,0)$, $B(35,5,0)$, $C(45,35,0)$, $A'(50,65,65)$, $B'(5,50,65)$, $C'(15,80,65)$, $K(70,55,60)$ and $M(15,25,5)$. Construct orthographic projections of the prism $ABCA'B'C'$ and line KM . ABC – base of the prism. Determine and construct points of intersection (piercing points) between prism $ABCA'B'C'$ and the line KM . Indicate visibility of the line segments.

15.5. Points $A(60,15,10)$, $B(10,55,40)$, $O(35,35,0)$. Construct orthographic projections of the cylinder of revolution and line AB . Radius of the cylinder base is 20mm. Cylinder base is located in the horizontal principal plane. O – center of the cylinder base. Determine and construct points of intersection (piercing points) between the cylinder and the line AB . Indicate visibility of the line segments.

15.6. Points $A(70,5,10)$, $B(70,65,10)$, $O(95,25,0)$, $O'(40,45,60)$. Construct orthographic projections of the cylinder of revolution and line AB . Radius of the cylinder base is 20mm. O – center of the cylinder base. OO' – axis of revolution. Determine and construct points of intersection (piercing points)

between the cylinder and the line AB. Indicate visibility of the line segments.

15.7. Points $A(75,60,35)$, $B(40,20,5)$, $O(75,25,0)$, $O'(30,60,50)$. Construct orthographic projections of the cylinder of revolution and line AB. Radius of the cylinder base is 20mm. O – center of the cylinder base. OO' – axis of revolution. Determine and construct points of intersection (piercing points) between the cylinder and the line AB. Indicate visibility of the line segments.

15.8. Points $A(55,45,15)$, $B(5,5,15)$, $O(30,30,0)$, $S(30,30,40)$. Construct orthographic projections of the cone of revolution and line AB. S – apex of the cone. Radius of the cone base is 20mm. O – center of the cone base. Determine and construct points of intersection (piercing points) between the cone and the line AB. Indicate visibility of the line segments.

15.9. Points $A(35,40,45)$, $B(35,40,5)$, $O(30,30,0)$, $S(30,30,40)$. Construct orthographic projections of the cone of revolution and line AB. S – apex of the cone. Radius of the cone base is 20mm. O – center of the cone base. Determine and construct points of intersection (piercing points) between the cone and the line AB. Indicate visibility of the line segments.

15.10. Points $A(55,45,5)$, $B(5,35,15)$, $O(30,30,0)$, $S(30,30,40)$. Construct orthographic projections of the cone of revolution and line AB. S – apex of the cone. Radius of the cone base is 20mm. O – center of the cone base. Determine and construct points of intersection (piercing points) between the cone and the line AB. Indicate visibility of the line segments.

15.11. Points $A(55,50,40)$, $B(5,5,40)$, $O(30,25,25)$. Construct orthographic projections of the sphere and line AB. Radius of the sphere is 20mm, O – center of the sphere. Determine and construct points of intersection (piercing points) between the sphere and line AB. Indicate visibility of the line segments.

15.12. Points $A(50,50,5)$, $B(5,25,20)$, $O(30,25,25)$. Construct orthographic projections of the sphere and line AB. Radius of the sphere is 20mm, O – center of the sphere. Determine and construct points of intersection (piercing points) between the sphere and the line AB. Indicate visibility of the line segments.

16. INTERSECTION BETWEEN SURFACES

For engineering modeling it's often necessary resolve positional problems - to determine relative position and mutual belongings of objects. This can be solved on the orthographical views of the objects. Basic positional problem: construct curve of intersection between two surfaces. To construct intersection of two surfaces means to define set of points which simultaneously belong to the both surfaces.

If one or two surfaces are perpendicular to principal planes (projecting surfaces), then positional problems can be solved without additional cutting surfaces.

Intersection between a cylinder and a prism. Define location of objects with respect to the principal planes: frontally perpendicular prism intersect horizontally perpendicular cylinder (Fig. 16.1).

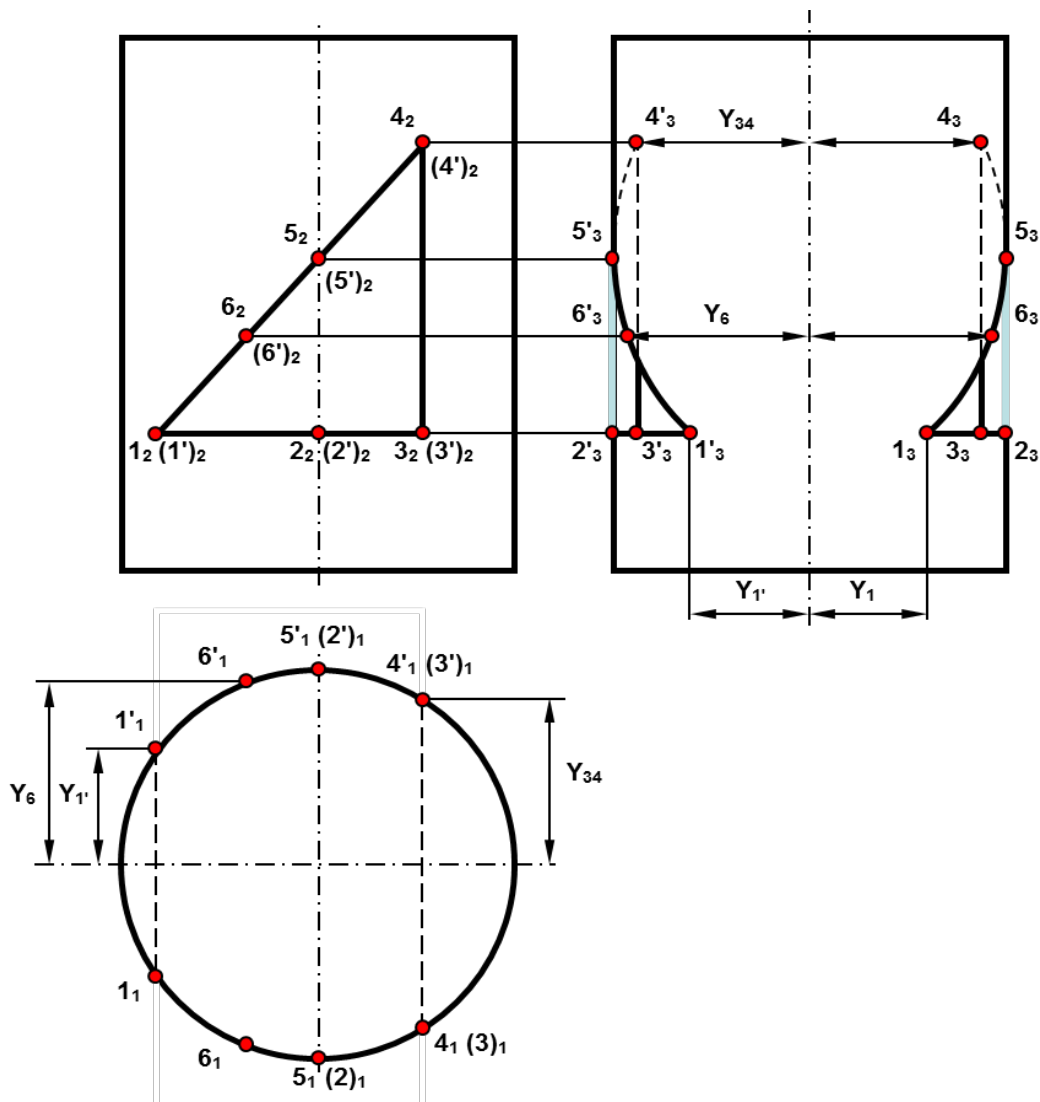


Fig. 16.1

Because the cylinder and the prism (see Fig. 16.1) are projecting figures (perpendicular to principal planes), projections of curves of intersection coincide to horizontal projection of the cylinder and frontal projection of the prism. Define location of the projections of the curve: horizontal projections are located between the points $1_1-6_1-5_1-4_1(3_1)$ and also between symmetric points, frontal projections are located between the points $1_2-6_2-5_2-4_2-3_2-2_2-1_2$ and also between proper symmetric points.

To construct profile projection of the curve of intersection measure proper y-coordinates of the points, which belong to the curve, and protract them on the profile principal plane on the proper projectors for each point. Then join the points in the necessary order and determine visibility of the curve segments.

To construct curve of intersection between surfaces we can use cutting surface method. As cutting surfaces we can use Cutting Planes or Cutting Spheres.

In the Fig. 16.2 two surfaces are cut by series of horizontal cutting planes. Series of m and n curves, obtained as a result of intersection between two surfaces and cutting planes, meet at the points M, N and K. Joining these points we can construct curve of intersection between two surfaces.

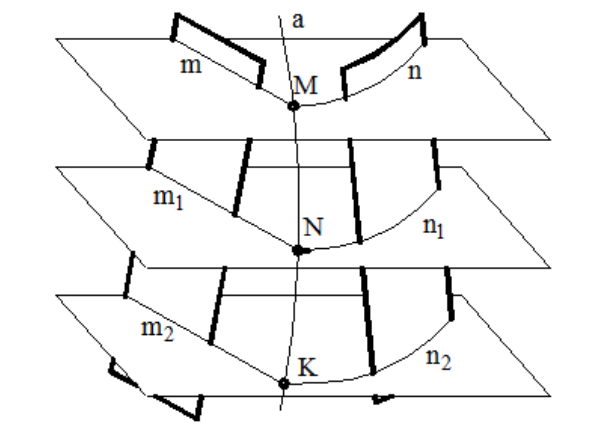


Fig. 16.2

Projection of a curve of intersection between surfaces lies in an area of intersection between projections of outlines of surfaces.

To construct curve of intersection it's desirable to indicate characteristic points — on the outline (upper, lower, left, right).

Intersection between a cone and a sphere. Cutting plane method (Fig. 16.3).

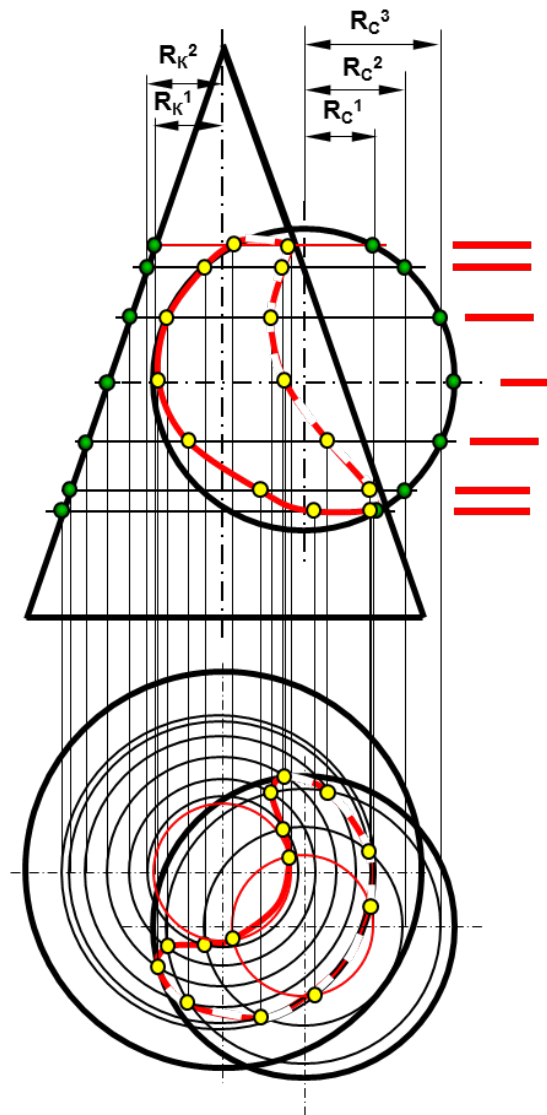


Fig. 16.3

Algorithm to construct curve of intersection between the surfaces using cutting plane method:

Determine area where curve of intersection can exist.

1. Determine surfaces of a model, perpendicular to principal planes.
2. Define base points for curve of intersection.
3. Assign type of cutting surfaces.
4. Define maximum number of cutting surfaces and their range.
5. Construct points of a curve of intersection.
6. Define base points where visibility of a curve changes.
7. Join all obtained points by a curve line in a proper sequence.

Problems

16.1. Points $A(60,15,0)$, $B(30,15,0)$, $C(15,35,0)$, $D(30,55,0)$, $E(60,55,0)$, $K(75,35,0)$, $K'(75,35,70)$, $M(85,25,20)$, $N(85,25,60)$, $L(85,65,40)$ and $L'(5,65,40)$. Construct orthographic projections of the horizontally perpendicular prism $ABCDEKA'B'C'D'E'K'$ ($ABCDEK$ – base of the prism, KK' – lateral edge) and profile perpendicular prism $MNLM'N'L'$ (MNL – base of the prism, LL' – lateral edge). Construct lines of intersection between the prisms.

16.2. Points $A(65,25,35)$, $B(30,75,35)$, $C(50,75,10)$, $A'(65,5,35)$, $S(40,40,65)$, $O(40,40,0)$. Construct orthographic projections of the frontally perpendicular prism $ABCA'B'C'$ (ABC – base of the prism, AA' – lateral edge) and cone of revolution. S – apex of the cone. Radius of the cone base is 30mm. O – center of the cone base. Construct lines of intersection between the prism and the cone.

16.3. Points $A(65,10,0)$, $B(5,10,0)$, $C(35,65,0)$, $S(35,35,60)$, $O(35,__,20)$. Frontally-perpendicular cylindrical hole intersects pyramid $SABC$. S – apex, ABC – base of the pyramid. Radius of the cylindrical hole is 15mm. O – center of the hole. Construct orthographic projections of the pyramid $SABC$ intersected by frontally-perpendicular cylindrical hole.

16.4. Points $S(40,40,65)$, $O(40,40,0)$, $C(50,__,25)$. Frontally-perpendicular cylindrical hole intersects cone of revolution. S – apex of the cone. Radius of the cone base is 40mm. O – center of the cone base. Radius of the cylindrical hole is 20mm. O – center of the hole. Construct orthographic projections of the cone of revolution intersected by frontally-perpendicular cylindrical hole.

16.5. Points $O(40,40,40)$, $A(70,__,40)$, $B(20,__,55)$ and $C(20,__,25)$. Frontally-perpendicular prismatic hole ABC intersects sphere. Radius of the sphere is 35mm, O – center of the sphere. Construct orthographic projections of the sphere intersected by frontally-perpendicular prismatic hole ABC .

16.6. Points $O(40,40,40)$, $A(65,30,__)$, $B(20,30,__)$, $C(20,60,__)$ and $D(45,60,__)$. Horizontally-perpendicular prismatic hole $ABCD$ intersects sphere. Radius of the sphere is 35mm, O – center of the sphere. Construct orthographic projections of the sphere intersected by horizontally-perpendicular prismatic hole $ABCD$.

16.7. Points $A(60,30,90)$, $A'(60,30,5)$, $B(15,30,75)$, $B'(105,30,15)$. Cylinder of revolution with the axis AA' and radius 25mm intersects cylinder of

revolution with the axis BB' and radius 20mm. Construct frontal view of the curve of intersection between this cylinders using cutting sphere method.

16.8. Points A(85,50,35), A'(5,50,35), S(45,50,90), O(45,50,0). Cylinder of revolution with the axis AA' intersects cone of revolution with the apex S and lower base center O. Radius of the cylinder is 25mm. Radius of the cone base is 45mm. Construct frontal view of the curve of intersection between the cylinder and the cone.

16.9. Points S(70,50,100), O(70,50,0), S'(10,50,5), O'(110,50,70). Construct curves of intersection between two cones of revolution. Cone one: S – apex, O – center, R = 40 mm – radius of the first cone. Cone two: S' – apex, O' – center, R' = 30 mm – radius of the second cone.

16.10. Points O(70,30,40), S(0,30,65). Horizontally perpendicular cylinder of revolution and a cone of revolution (S – apex of the cone) are circumscribed about a sphere (O – center of the sphere, R = 20 mm – radius of the sphere). Construct frontal projection of the curves of intersection between the cone and the cylinder.

16.11. Point O(45,40,30). Horizontally and profily perpendicular cylinders of revolution are circumscribed about a sphere (O – center of the sphere, R = 15 mm – radius of the sphere). Construct frontal projection of the curves of intersection between the cylinders of revolution.

17. DEVELOPMENTS

Some curved surfaces can be developed so that they coincide completely (with all their points) with a plane without stretching or shrinking. Obtained planar figure is called development of a surface while surfaces are called **developable**.

They comprise polyhedrons and ruled surfaces where adjacent rectilinear generatrices are parallel or intersect, or are tangent to sphere curve.

All double-curved surfaces and the ruled surfaces which cannot be developed into a plane are called **nondevelopable** (or *warped*) surfaces.

Properties of developments (Fig. 17.1):

1. Each point on the development corresponds to a single point of a surface.
2. Straight lines of a surface remain straight on a development.
3. Straight line segments preserve their lengths.
4. An angle formed by lines on a surface remains equal to an angle between the corresponding lines on the development.
5. The area of a closed domain on a surface retains its magnitude within the corresponding closed domain on the development.
6. Shortest distance (beeline) on a surface develops into a straight line on a development (developed distance).
7. Parallel lines develop to parallel lines.

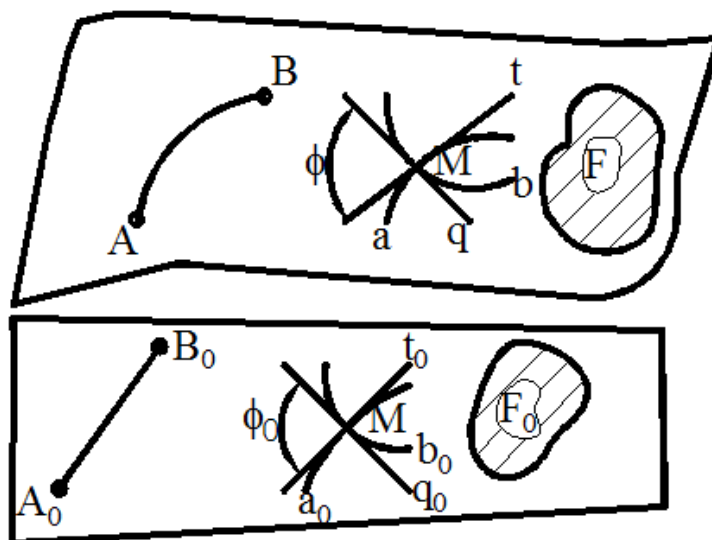


Fig. 17.1

Triangulation method

Development of a pyramid. Lateral faces of a pyramid are triangles (Fig. 17.2). To find true sizes of triangles you can use transformation of a drawing or find true lengths of corresponding edges (Fig. 17.2, 17.3). Because base of a pyramid is parallel to Π_1 -plane, it's sufficient to find true lengths of lateral edges AS, BS, CS. After that construct, for example, BCS-face, and supplement construction adding ASB and ASC faces.

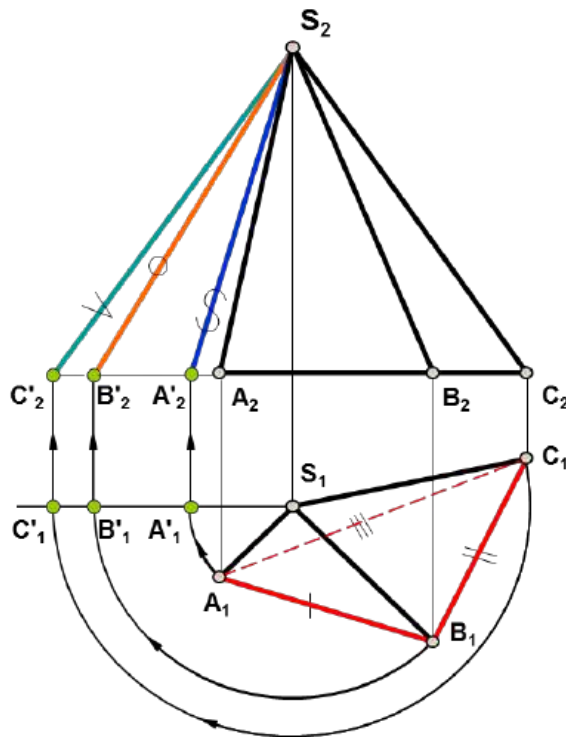


Fig. 17.2

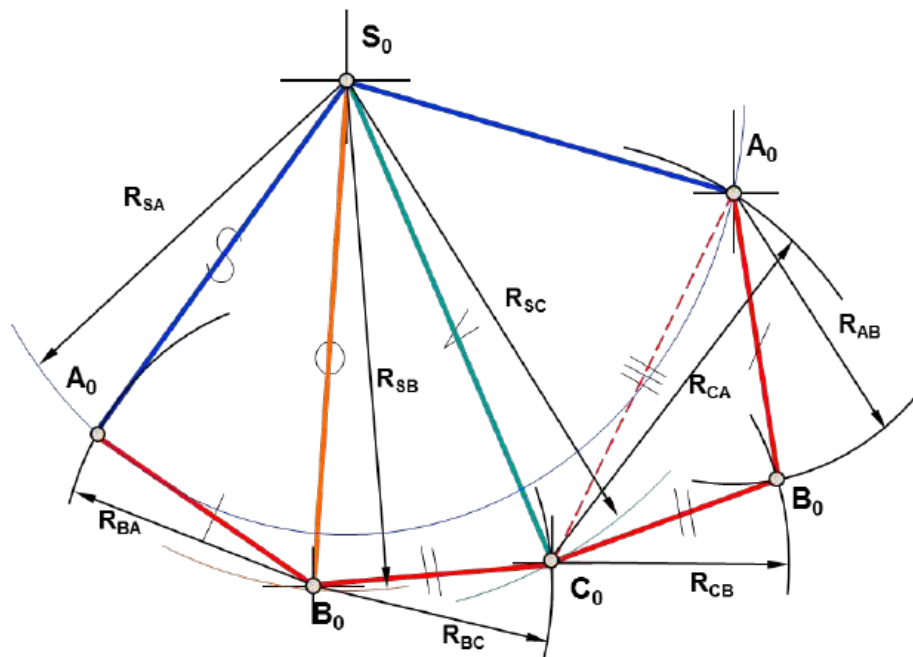


Fig. 17.3

Stretch out Line method

Development of a prism. This method is used when generatrices are parallel to a principal plane of projection (Fig. 17.4).

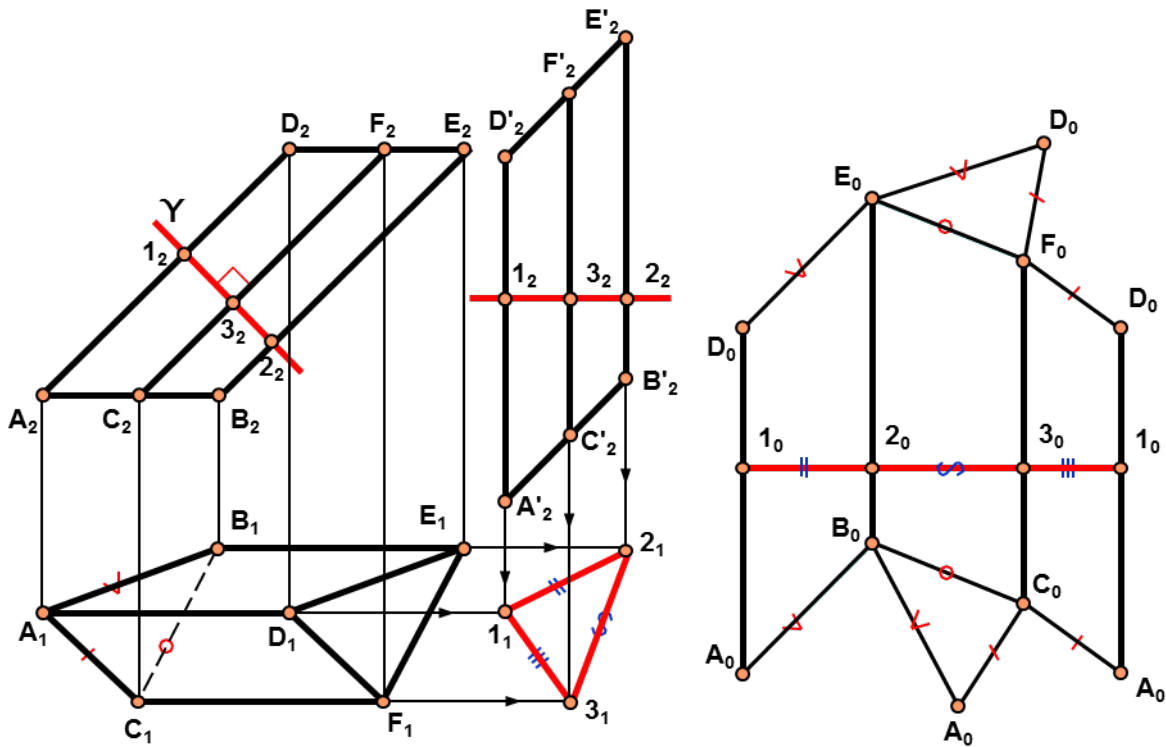


Fig. 17.4

Cut ABCDEF-prism by the plane γ , perpendicular to the lateral edges of prism. Construct section of the prism by the plane in question – $\Delta 123$. Define true lengths of sides of 123-triangle. On a free space of a drawing draw horizontal straight line a . From the arbitrary point 1_0 on this straight line protract line segments $[1_0 2_0]$, $[2_0 3_0]$, $[3_0 1_0]$, equal to the sides of 123-triangle. Through the points 1_0 , 2_0 , 3_0 , 1_0 draw straight lines, perpendicular to the a -line, and protract from the points 1_0 , 2_0 , 3_0 , 1_0 line segments, equal to lengths of corresponding lateral edges ($[1_2 A_2]$, $[1_2 D_2]$, $[2_2 B_2]$, $[2_2 E_2]$, $[3_2 C_2]$, $[3_2 F_2]$), then join obtained points $A_0 B_0 C_0 A_0$ and $D_0 E_0 F_0 D_0$ line segments. Obtained planar figure $A_0 B_0 C_0 A_0 D_0 E_0 F_0 D_0$ is a development of a lateral surface of the prism.

To construct full development of a prism supplement obtained development by true figures of top and bottom bases.

Problems

17.1. Points $S(5,30,45)$, $A(60,20,0)$, $B(45,35,0)$, $C(25,5,0)$, $K(25,20, _)$. Pyramid SABC is intersected by frontally-perpendicular plane, which passes through OY-axes at an angle 45° to the horizontal principal plane. Construct development of the pyramid SABC, truncated by the frontally-

perpendicular plane. On the development of the pyramid SABC draw the point K, which belongs to the pyramid surface. Projection K_1 is invisible.

17.2. Points A(70,20,5), B(50,5,5), C(40,30,5), A'(35,20,45), K(40,__,30), M(30,0,0). Prism ABCA'B'C' is intersected by horizontally-perpendicular plane, which passes through the point M at an angle 45° to the frontal principal plane. Construct development of the prism ABCA'B'C', truncated by the horizontally-perpendicular plane. Convenient to use stretch-out line method. On the development of the prism ABCA'B'C' draw the point K, that belongs to the prism surface. Projection K_2 is invisible.

17.3. Points O(30,25,0) and O'(80,25,30). OO' – axis of oblique cylinder. O – center of the lower base, located in horizontal principal plane. Radius of the lower base of the cylinder is 20mm. O' – center of upper base, which is located in profile principal plane. Construct development of the lateral surface of the cylinder.

17.4. Points S(0,60,50) and O(45,25,0). Oblique cone is intersected by the plane $X=10$. S – apex of oblique cone. Base of the cone is a circle, located in horizontal principal plane. O – center of the circle. Radius of the circle is 25mm. Construct development of the lateral surface of the cone. Draw a curve of intersection on the development of the cone.

17.5. Construct development of a sphere of a radius 40 mm.

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